

A Position Domain Cross-Coupled Iteration Learning Control for Contour Tracking in Multi-axis Precision Motion Control Systems

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Abstract. A novel cross-coupled iteration learning controller in position domain is presented to improve contour tracking performance for multi-axis micro systems executing repetitive tasks. The position domain iteration learning control (PDILC) is combined with position domain cross-coupled control (PDCCC) to develop a position domain cross-coupled iteration learning control (PDCCILC). The stability and performance analysis are given based on lifted system representation in time domain. To illustrate effectiveness and good tracking performance of the proposed control method, simulation studies are conducted based on an identified model of a three dimensional micro-motion stage.

Keywords: Position domain · CCILC · Contour tracking · Multi-axis motion · Precision motion control

1 Introduction

High precision manufacturing processes and micromanipulation have produced a need for increased research in precision motion control (PMC). In multi-axis PMC system like 3D printer, micro-assembling, nano-lithography and so on, contour tracking is one of the crucial control problems. Many control strategies have been developed to improve the tracking performance of each individual axis motion, such as proportional integral derivative (PID) controller [1], robust control [2], sliding-mode control [3], iterative control [4], repetitive control [5,6], polynomial-based pole placement control [7] and so on. For decoupled multi-input multi-output (MIMO) control systems, a traditional control regards MIMO system as many single-input single-output (SISO) systems designed each axis separately regardless of other axes. However, a good tracking performance for each individual axis does not guarantee the reduction of contour errors for a multi-axis motion system, as poor synchronization of relevant motion axes may result in diminished accuracy of the contour tracking performance [8].

Contour error is defined as an orthogonal component of the derivation of an actual contour from the desired one [8]. To improve the contour performance, cross-coupling control (CCC) was developed by Koren [9]. CCC utilizes coupling gains to

couple the individual axis errors of SISO systems together and applies a controller to the combined signal. CCC has been used in multi-axis motions for manufacturing, especially in computerized numerical control (CNC). It should be mentioned that, in CCC, each axis still needs to be controlled, and there still have some tracking errors for each individual axis motion which will affect the final contour tracking performance [8]. Another method to decrease contour errors is event-driven control [10], which triggers the controller to update a control action through an event, such as a new measurement or distance information. The major drawback of event-driven control is the difficulty in developing system theory and performance sacrifice. Besides, Ouyang proposed a novel PID feedback controller based on the position domain (PDPID) which perceives motion system as a master-slave cooperative system to guarantee synchronization and improve the contour tracking performance. The method has been applied on CNC [11] and robotic system [12]. However, for those manufacturing systems to perform repetitive task, the feedback controller alone cannot decrease the repetitive contour error [13].

To improve repetitive performance, iterative learning control (ILC) can be implemented because of the repetitive feature. ILC allows the controller to learn from previous executions (trials, iterations, passes) to achieve better performance [14]. Barton [13], [15,16] combined cross-coupled iterative learning control (CCILC) with individual axis ILC to improve both individual axis and contour tracking performance. However, for multi-axis systems, no matter CCILC or individual axis ILC, the controllers are designed in time domain where reference trajectories are designed as a function of time, which may not achieve good motion synchronization.

Therefore the main goal of this paper is to provide a new method for multi-axis PMC to improve repetitive contour tracking performance by introducing position domain method into CCILC design. The novel position domain CCILC (PDCCILC) controller is advantageous on maintaining multi-axis synchronization with reducing individual axis and contour error simultaneously when compared with existing time domain CCILC.

The outline of this paper is as follows. Section II gives a brief introduction of the controller design background about ILC, CCC and PDC. In section III, PDCCILC control law is proposed and analyzed. The simulation results and comparison between time domain CCILC (TDCCILC) and PDCCILC are presented in Section IV. Conclusions are given in Section V.

2 Controller Design Background

Cross-coupled iteration learning control is a combination of traditional feedback CCC and feedforward ILC. Its main advantage is improving contour tracking performance in multi axis precision motion systems. Before discussing position domain CCILC, the following sections briefly introduce ILC, CCC and PDC.

2.1 ILC

ILC is firstly proposed by Uchiyama in 1978 [17] and widely discussed in [14], [18] and so on. As an intelligence algorithm, it is advantageous on achieve high tracking precision when a system executes repetitive tasks.

Considering a discrete LTI and SISO system

$$Y_j(z) = P(z)U_j(z) + D(z) \tag{1}$$

where z stands for the z -transformation of a system, j is the iteration index, $Y_j(z)$ is the output, $U_j(z)$ is the control signal, $D(z)$ is the exogenous signal and $P(z)$ is the transfer function of system.

A widely used control law of ILC is Eq.(2) and the ILC system is asymptotically stable (AS) if Eq.(3) can be satisfied.

$$U_{j+1}(z) = Q(z)[U_j(z) + L(z)E_j(z)] \tag{2}$$

$$\|Q(z)[1 - L(z)P(z)]\|_\infty < 1 \tag{3}$$

where $Q(z)$ is a filter, $L(z)$ is learning function, tracking error $E_j(z) = Y_d(z) - Y_j(z)$ ($Y_d(z)$ is the desired output), and $\|\cdot\|_\infty$ is the infinite norm of the matrix.

2.2 CCC and CCILC

In some multi-axis systems, prime concern should be emphasized in contour error rather than separate axis tracking error [19]. Cross coupled control is a technique to reduce contour error by choosing appropriate coupling gains and coordinating the motion of two coupled axis.

Determining coupling gains is vital in CCC as they are used to calculate contour error and allocated control signal to individual axis. In linear XY plane contour tracking, contour error ε is defined as Eq.(4)

$$\varepsilon = -C_x e_x + C_y e_y \tag{4}$$

where $C_x = \sin\theta$, $C_y = \cos\theta$, θ is the angle between the x-axis and the desired linear trajectory, e_x and e_y are x-axis and y-axis tracking error. For a circular contour tracking, $C_x = \sin\theta - e_x/2R$, $C_y = \cos\theta + e_x/2R$, where R is the radius and θ is angle between the x-axis and the tangent of the desired tracking point in the circle. A complex contour can be achieved by combining a series of linear and circular parts together.

When associating ILC with CCC, the CCILC algorithm is approached. The general CCILC control structure is presented in Fig. 1 [13],[15]. A novel control law combining individual axis ILC algorithm for x-axis and y-axis with CCILC law is given in [16] as

$$\begin{bmatrix} U_x \\ U_y \end{bmatrix}_{j+1} = Q \left(\begin{bmatrix} U_x \\ U_y \end{bmatrix} + \begin{bmatrix} L_x & 0 \\ 0 & L_y \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_c \end{bmatrix} \right) \tag{5}$$

where U , L , E , and C are control signal, learning function, individual axis tracking error and coupling gain matrix respectively, x , y and c represent x-axis, y-axis and contour.

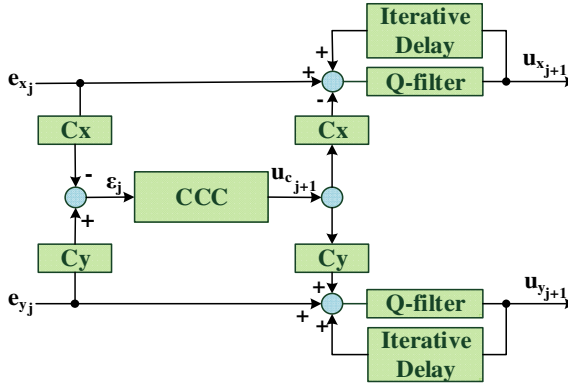


Fig. 1. CCILC control structure

2.3 PDC

Being different from the aforementioned control methods implemented in time domain, a position domain controller focuses on improving synchronization of relevant individual axis motion in multi-axis systems. A PD-type contour tracking control law established in position domain were proposed in [20].

In a two DOF decoupled parallel motion system, a PD-type feedback control signal $U_y(x)$ of y-axis (slave motion) in position domain is related to x-axis position (master motion), which can be expressed as

$$U_y(x) = K_{py}e_y(x) + K_{dy}e'_y(x) \tag{6}$$

where K_{py} and K_{dy} are proportional gain and differential gain, $e_y(x) = y_d(x) - y(x)$, $e'_y(x) = y'_d(x) - y'(x)$. It can be seen that position domain PD law uses x-axis position as a reference rather than time. Then, to achieve accurate contour performance, a high precision measurement is required in the master motion direction.

Convert Eq.(6) to time domain Eq.(8) using Eq.(7).

$$\dot{y}(t) = dy/dt = (dy/dx) \cdot (dx/dt) = y'(x) \dot{x}(t) \tag{7}$$

$$U_y(t) = K_{py}(y_d(t) - y(t)) + K_{dy}(\dot{y}_d(t) - \dot{y}(t)) / \dot{x}(t) \tag{8}$$

3 Position Domain CCILC

We here provide a novel method for multi-axis manufacturing systems to improve repetitive contour tracking performance by introducing position domain method into CCILC design.

3.1 Control Law

Considering a two input two output, linear time invariant (LTI) system, the PDCCILC control signal of x-axis and y-axis can be given as

$$u_{x_{j+1}}(t)=Q[u_x(t)+L_x e_x(t)-C_x L_e \varepsilon(x)]_j \tag{9}$$

$$u_{y_{j+1}}(x)=Q[u_y(x)+L_y e_y(x)+C_y L_e \varepsilon(x)]_j \tag{10}$$

where j is the iteration index, u is the control signal, Q is a filter, L is learning function and ε is contour error. Eq.(9) is a time domain law except the last term, which can be regard as a sacrifice of x-axis accuracy for contour tracking performance. We here mainly discuss the control law of y-axis as it is completely in position domain.

Applying PID type ILC and PD type CCC, Eq.(10) then can be written as

$$u_{y_{j+1}}(x)=Q[u_y(x)+K_{py} e_y(x)+K_{iy} \int_0^{\Delta s} e_y(x)dx +K_{dy} e_y'(x) +C_y(K_p^{xy} e_\varepsilon(x)+K_d^{xy} e_\varepsilon'(x))]_j \tag{11}$$

3.2 Stability

For linear trajectory, substituting Eq.(4) into Eq.(11), yielding Eq.(12).

$$u_{y_{j+1}}(x)=Q[u_y(x)+K_{py} e_y(x)+K_{iy} \int_0^{\Delta s} e_y(x)dx +K_{dy} e_y'(x) +C_y K_p^{xy} C_y e_y(x)-C_y K_p^{xy} C_x e_x(x)+C_y K_d^{xy} C_y e_y'(x)-C_y K_d^{xy} C_x e_x'(x)]_j \tag{12}$$

Convert Eq. (12) to time domain Eq. (13).

$$u_{y_{j+1}}(t)=Q[u_y(t)+K_{py} e_y(t)+K_{iy} \int_0^{\Delta t} e_y(t)\dot{x}(t)dt +K_{dy} (\dot{e}_y(t)/\dot{x}(t)) +C_y K_p^{xy} C_y e_y(t)-C_y K_p^{xy} C_x e_x(t)+C_y K_d^{xy} C_y (e_y'(t)/\dot{x}(t))-C_y K_d^{xy} C_x (e_x'(t)/\dot{x}(t))]_j \tag{13}$$

According to Eq. (13), numerical error of control signal may occur when x-axis under a low speed because the denominator $\dot{x}(t)$. To avoid this problem, an increment of x-axis position Δx in time Δt is applied.

$$\int_0^{\Delta t} e_y(t)\dot{x}(t)dt= \Delta x(e_y(t)+e_y(t-\Delta t))/2 \tag{14}$$

$$e_y \dot{(t)}/\dot{x}(t) = (e_y(t) - e_y(t - \Delta t)) / \Delta x \tag{15}$$

$$e_x \dot{(t)}/\dot{x}(t) = (e_x(t) - e_x(t - \Delta t)) / \Delta x \tag{16}$$

Substituting Eq.(14) - (16) into Eq.(13) yielding,

$$u_{y_{j+1}}(t) = Q[u_y(t) + \alpha \cdot e_y(t) + \beta \cdot e_y(t - \Delta t) + \gamma]_j \tag{17}$$

where

$$\alpha = K_{py} + K_{iy} \Delta x / 2 + K_{dy} / \Delta x + C_y^2 K_d^{xy} / \Delta x \tag{18}$$

$$\beta = K_{iy} \Delta x / 2 - K_{dy} / \Delta x - C_y^2 K_d^{xy} / \Delta x \tag{19}$$

$$\gamma = -C_y K_d^{xy} C_x (e_x(t) - e_x(t - \Delta t)) / \Delta x - C_y K_p^{xy} C_x e_x(t) \tag{20}$$

According to Eq. (17), there is time delay in control law. To analysis the definitive stability and convergence in time domain, the lifted-system representation is applied.

$$y_j(k) = P(q)u_j(k) + d(k) \tag{21}$$

$$e_j(k) = y_d(k) - y_j(k) \tag{22}$$

where k is the discrete time index, q is the forward time-shift operator ($qx(k) \equiv x(k+1)$).

Using Eq.(21) and Eq.(22), Eq.(17) can be written as Eq.(23).

$$u_{y_{j+1}}(k) = Q[(I - \alpha P_y - \beta P'_y)u_{y_j}(k) + (\alpha + \beta q^{-\Delta k})(y_d(k) - d(k)) + \gamma'] \tag{23}$$

where $P'_y(q) = q^{-\Delta k} \cdot P_y(q)$, $q^{-\Delta k}$ stands for time delay with discrete step length of Δk . γ' is independent of convergence of y-axis control signal because it is a term related to e_x rather than u_j .

Then, system is asymptotically stable (AS) if there exists appropriate parameters α and β satisfying

$$\rho(Q(I - \alpha P_y - \beta P'_y)) < 1 \tag{24}$$

where ρ is the spectral radius of the matrix. The parameter α and β are bounded on condition that the trajectory and motion of x-axis are planned appropriate with the position increment Δx bounded.

3.3 Performance

If the AS condition Eq.(24) is satisfied, the performance of system under this law is based on the asymptotic value of the error.

$$u_{y_{\infty}}(k) = Q[(\alpha + \beta q^{-\Delta k})(y_d(k) - d(k)) + \gamma'] / (I - Q(I - \alpha P_y - \beta P'_y)) \tag{25}$$

$$\begin{aligned}
 e_{y,\infty}(k) &= \lim_{j \rightarrow \infty} (\mathbf{y}_d(k) - \mathbf{P}_y \mathbf{u}_{y_j}(k) - \mathbf{d}(k)) \\
 &= \mathbf{y}_d(k) - \mathbf{d}(k) - \mathbf{P}_y \mathbf{Q} [(\alpha + \beta q^{-\Delta k}) \cdot (\mathbf{y}_d(k) - \mathbf{d}(k)) + \gamma'] / (\mathbf{I} - \mathbf{Q}(\mathbf{I} - \alpha \mathbf{P}_y - \beta \mathbf{P}_y')) \quad (26)
 \end{aligned}$$

where \mathbf{Q} filter is designed to determine which frequencies are emphasized in the learning function. Term γ' indicates that tracking performance in y-axis is associated with x-axis position accuracy. The contour error ε can be calculated using Eq.(4).

4 Evaluation

4.1 Micro-motion Stage

For this paper, three-dimensional translational DOF micro-motion stage was used as a control case of multi-axis micromanipulator contour tracking. As shown in Fig. 2, each motion direction of the stage is driven by a piezoelectric actuator, and the motion is transferred by a compliant joint to the moving platform where an end-effector can be set up.

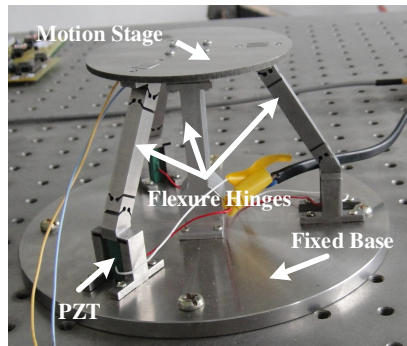


Fig. 2. Micro-motion stage

4.2 Control Structure

The control structure in this case was built as Fig. 3, a feedforward PDCCILC controller was adopted to decrease individual axis and contour errors, combining with a feedback H_∞ controller to improve system robustness.

4.3 Simulation Cases and Results

In this part, comparison studies between time domain ILC (TDILC) and position domain ILC (PDILC), time domain CCILC (TDCCILC) and position domain CCILC (PDCCILC) were made to verify the effectiveness of position domain design method and the proposed PDCCILC controller.

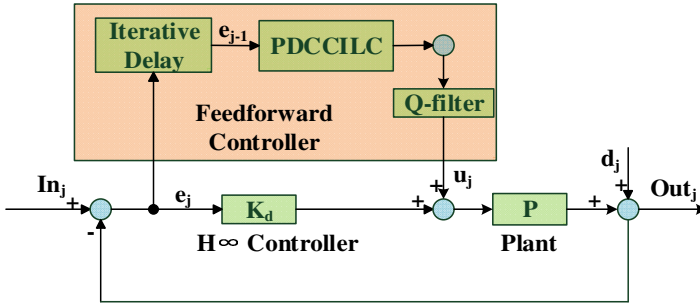


Fig. 3. Feedback and feedforward control structure

Two types of three dimension reference trajectories shown in Fig. 4 (zigzag motion and quadrangle motion) were adopted as examples to show contour tracking performance in linear motions. In ILC iterations, the initial conditions should be set the same, so the red marked point in Fig. 4 is set as the home position.

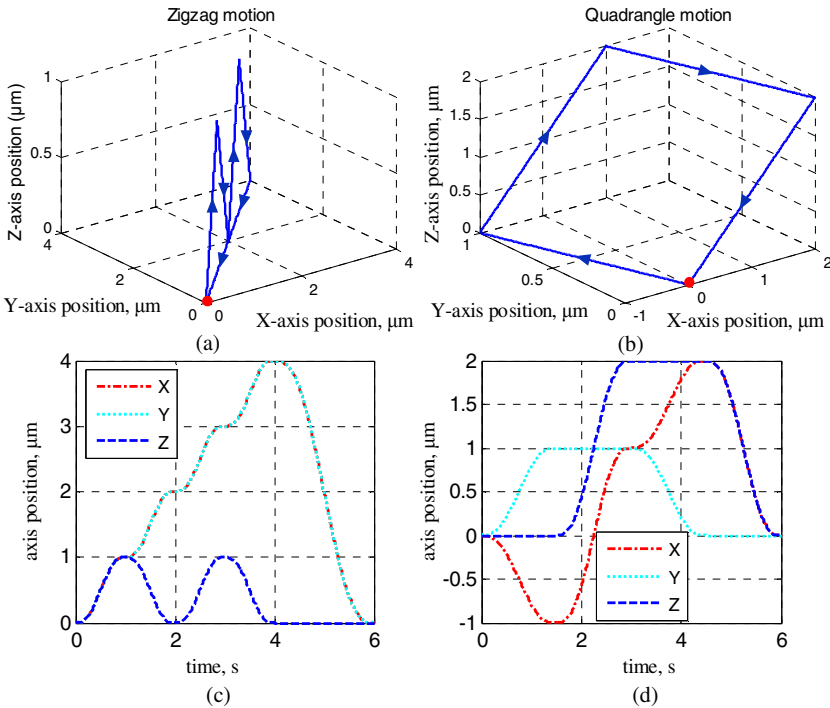


Fig. 4. Reference trajectories. (a) Zigzag trajectory. (b) Quadrangle trajectory. (c) Zigzag axis motion versus time. (d) Quadrangle axis motion versus time.

Base on system AS condition in Eq. (24), the PID-type ILC gains and PD-type CCC gains were selected. For comparison, all of the gains for the four type controllers were set the same (shown in Tab.1) and iteration was set 300 times.

Table 1. Control gains used in four controllers

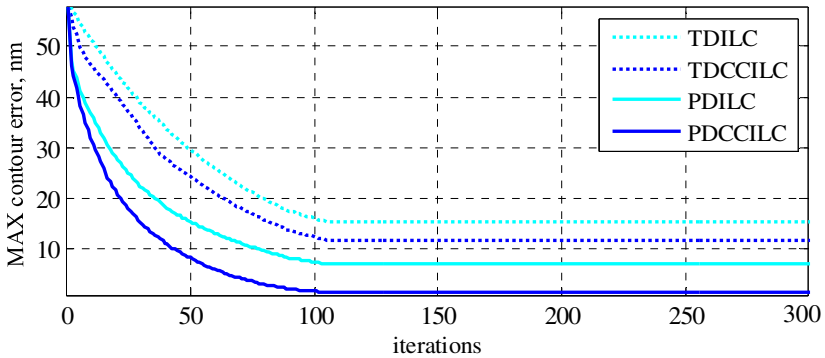
Reference	Axis gain in ILC			Contour gain in CCC	
	K_p	K_i	K_d	K_p	K_d
Zigzag motion	1	0.3	0.3	0.4	0.4
Quadrangle motion	0.3	2	0.3	0.4	0.4

Zigzag Motion Tracking Case

As the x-axis and y-axis trajectory is the same, we here choose x-z plane to evaluate contour tracking performance for aforementioned four controllers. Fig. 5 shows MAX contour errors of x-z plane in iteration process. Errors under the four controllers are uniformly convergent from the 100th iteration to the last, which indicates the asymptotic stable condition is satisfied (Eq.(24) for the proposed PDCCILC).

Compared with time domain controllers, position domain controllers are superior to time domain controllers under same parameters and external disturbance.

For CCILC, the maximum steady error of PDCCILC is smaller (about 1.53nm), which is a 97% decrease of the initial error (58.36nm) and is 13% of MAX contour error of TDCCILC (11.9 nm).

**Fig. 5.** MAX error of x-z plane zigzag contour tracking

Contour tracking in x-z plane for the four controllers is shown in Fig. 6. Part a and part b are respectively the amplified linear and angular contour of reference. In both parts, position domain controllers prove to be better in following reference than time domain ones, and PDCCILC is better than TDCCILC, which keeps correspondence with analysis in Fig. 5. More specific supporting data about MAX and root mean square (RMS) contour errors can be found in Tab.2.

Table 2. MAX and RMS contour errors of the last iteration

Controller	Zigzag Motion (nm)			Quadrangle Motion (nm)			
		XY	XZ	YZ	XY	XZ	YZ
TDILC	RMS	0.01	6.21	6.21	3.41	9.02	7.58
	MAX	0.06	15.49	15.49	6.89	19.5	19.5
TDCCILC	RMS	0.01	1.43	1.43	1.62	3.77	3.26
	MAX	0.04	11.90	11.90	4.03	8.50	8.50
PDILC	RMS	0.00	0.11	0.11	1.60	1.97	2.23
	MAX	0.01	7.11	7.11	6.24	4.91	3.58
PDCCILC	RMS	0.00	0.02	0.02	0.86	1.02	1.17
	MAX	0.00	1.53	1.53	2.20	2.88	1.84

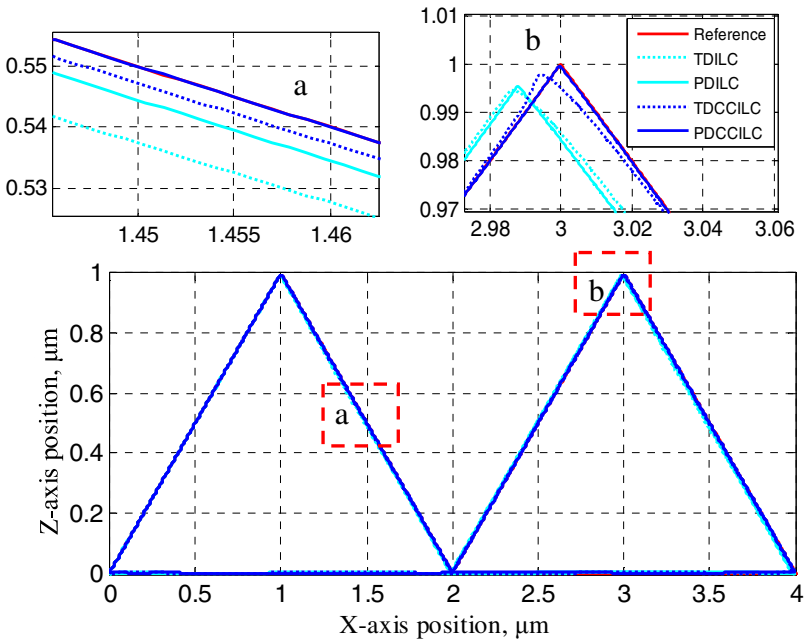


Fig. 6. Zigzag contour tracking in x-z plane of four controllers in the last iteration

Quadrangle Motion Tracking Case

In this example, 3-dimensional quadrangle motion was used to evaluate the four controllers. Fig. 7 shows the MAX contour error in iteration process. Fig. 10 shows contour tracking in x-z plane for the four controllers. (The x-y and y-z plane is omitted for brief.)

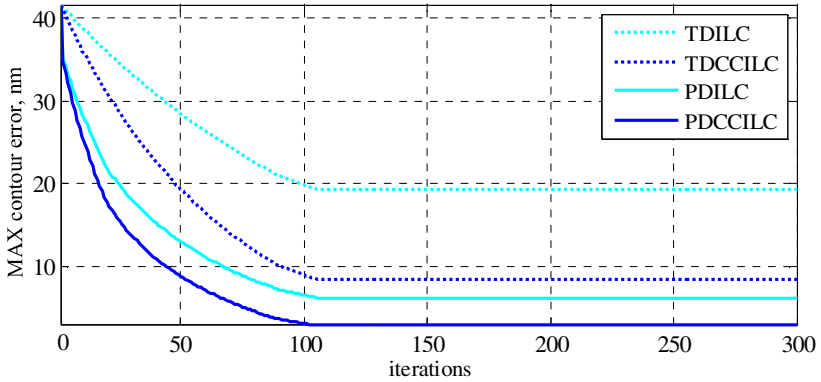


Fig. 7. MAX error of x-z plane quadrangle contour tracking

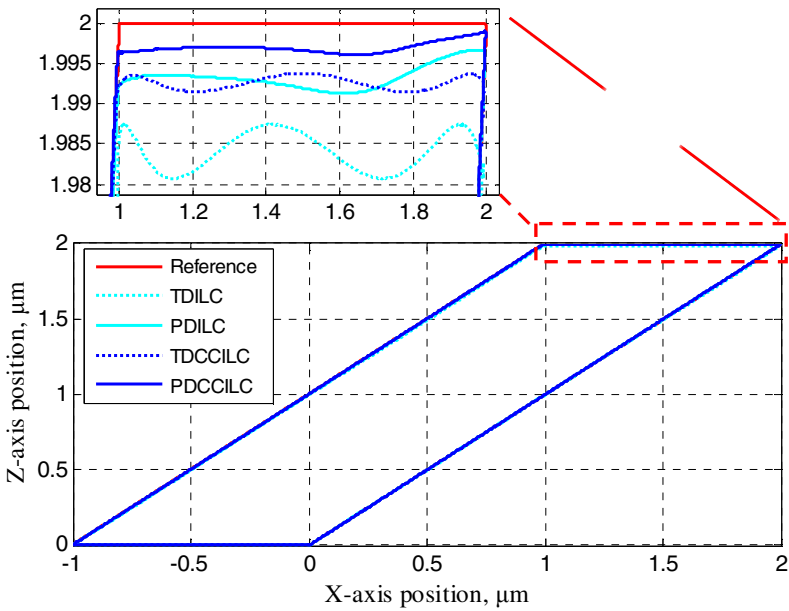


Fig. 8. Quadrangle contour tracking in x-z plane of four controllers in the last iteration

In Fig. 7, PDILC achieves better performance than TDILC (maximum steady error 4.91nm versus 19.5nm). On the other hand, MAX error under PDCCILC control is decreased by 93% from 41.6nm and keeps steady at 2.88nm, which is 34% of TDCCILC. Fig. 8 demonstrates the best contour tracking performance of the proposed PDCCILC, which is in agreement with Fig. 5 and Fig. 6.

5 Conclusions

This paper has presented a novel control law of CCILC designed in position domain for multi-axis precision motion control systems. The stability and performance analysis of the proposed PDCCILC were conducted using lifted system representation method. To evaluate the control law, simulations were performed based on an identified model of a three axis micro-motion stage. Four controllers under the same control gains and external disturbance were designed to make comparisons and to demonstrate the superiority of PDCCILC. Simulation results proved that PDCCILC decreased contour error significantly and achieved the best tracking performance among the four controllers.

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