Hysteresis modelling and feedforward compensation of piezoelectric nanopositioning stage with a modified Bouc-Wen model

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Piezoelectric actuators (PEAs) are widely applied in various nanopositioning equipment. However, the strong hysteresis nonlinearity compromises the positioning accuracy. In this work, a novel modified Bouc-Wen (MBW) model with a polynomial function of the differential of the input is established for modelling the hysteresis nonlinearity of the PEA-actuated nanopositioning stages. The particle swarm optimisation algorithm is adopted to identify the parameters of the MBW model with a set of input–output experimental data. The obtained model with the corresponding identification parameters matches well the experimental data with 0.31% relative error. A feedforward compensator based on the obtained model is also applied to compensate the hysteresis nonlinearity. Experiments are conducted to validate the effectiveness of this approach, and the results show the great improvement of positioning accuracy of the stage.

1. Introduction: Piezoelectric actuators (PEAs) have been extensively applied in various nanopositioning equipment such as nanomanipulators [1], scanning probe microscopy [2] and piezo-actuated flexure stages [3] because of their small size, high positioning resolution and quick frequency response [4]. However, the inherent hysteresis nonlinearity of PEAs strongly compromises the accuracy of positioning [5–7]. Therefore, it is necessary to compensate the hysteresis so that the relationship between the output displacement and the input control signal is linear.

In recent literatures, a number of control methods have been developed to achieve this goal, including hysteresis-observer [8–9], iterative control approach [10–12], robust control [13], H-∞ robust disturbance observer [14] and sliding mode control [15–17]. These approaches can be roughly classified into feedforward control, feedback control and feedforward–feedback control. The feedforward control approach is most commonly used and effective [18], which is based on different mathematical models to characterise the hysteresis. Many efforts have been made to develop sufficiently accurate mathematical model and design feedforward controller for hysteresis compensation. As a consequence, a number of hysteresis models are proposed, such as the Preisach model [19–21], the Prandtl-Ishlinskii model [22–24], the Dahl model [25] and the Bouc-Wen (BW) model [5, 26–27].

Among these models, the BW model can match the behaviour of a wide class of hysteresis system. It has been extensively applied in piezoelectric hysteresis modelling with the advantage of simplicity of computation and implementation, because it is only based on one differential equation with a few parameters [28]. However, the symmetrical hysteresis of the BW model will result in large modelling error when modelling the PEAs with asymmetrical hysteresis. For characterising the asymmetrical hysteresis, the asymmetrical component is introduced into the BW model in [6], and a non-odd input function in [29] or a third-order input function in [30] is utilised to develop the asymmetrical BW model. Although all the three modifications realise asymmetrical characteristics and reduce modelling errors to some degree, there are still large modelling errors especially at the minimum and maximum values of the input signal. So it is difficult to achieve good performance through feedforward compensation control. Hence, it is meaningful to further research on modelling the hysteresis more accurately with the BW model and design corresponding feedforward compensation controller based on the established model.

The primary goal of this research is to improve the positioning accuracy of the PEA-actuated stage utilising a hysteresis model and a model-based controller. The novelty of this work is that the proposed modified Bouc-Wen (MBW) model introduces the asymmetrical component and the polynomial of the differential of the input simultaneously to characterise the hysteresis nonlinearity accurately. The particle swarm optimisation (PSO) algorithm is adopted to identify the parameters of the MBW model with a set of input–output experimental data to and from the stage. The symmetrical BW model in [5] and the asymmetrical BW model in [6] are also identified for comparison. Then, model-based controllers based on the three models are designed and applied to reduce the hysteresis of the stage. As the same direct model is used in the compensator, no more computation is required for the compensator. The performance of the proposed compensator based on the MBW model is experimentally verified. And it is obvious that the performance with the MBW model is better than the other two.

The remainder of this Letter is organised as follows. The hysteresis analysis and classical BW (CBW) model are presented in Section 2. Section 3 presents the MBW model and parameters identification method. Section 4 is dedicated to design the feedforward controller based on the established model. Finally, experimental results and validation are presented in Section 5 and Section 6 draws the conclusion.

2. Hysteresis analysis: The experimental setup is developed and shown in Fig. 1. According to Fig. 1, the setup is composed of a three-axis nanopositioner (P-561.3CD), the data acquisition card (PCI 6289, National Instrument), a piezo amplifier module (E-503.00, Physik Instrumente) with a fixed gain of 10, a sensor monitor (E-509.C3A, Physik Instrumente), the target PC and development PC. When conducting experiments, only the x-axis with a stoke of 100 µm is adopted. The control input voltage range is 0–10 V. The output voltage range is 0–10 V, which is normalised with respect to 0–100 µm. The control system of the nanopositioning stage is developed based on Simulink Real-Time in MATLAB/Simulink environment. The control algorithm is designed in Matlab/Simulink block diagram on develop PC, and executed in real time on the target PC (CPU: Intel Core i5 @3.3 GHz) after compiling. In this Letter, the sample rate is set to 2 kHz.

In a PEA, the actual input–output relationship between the applied voltage and the output displacement is nonlinear with the asymmetric property because of the inherent hysteresis

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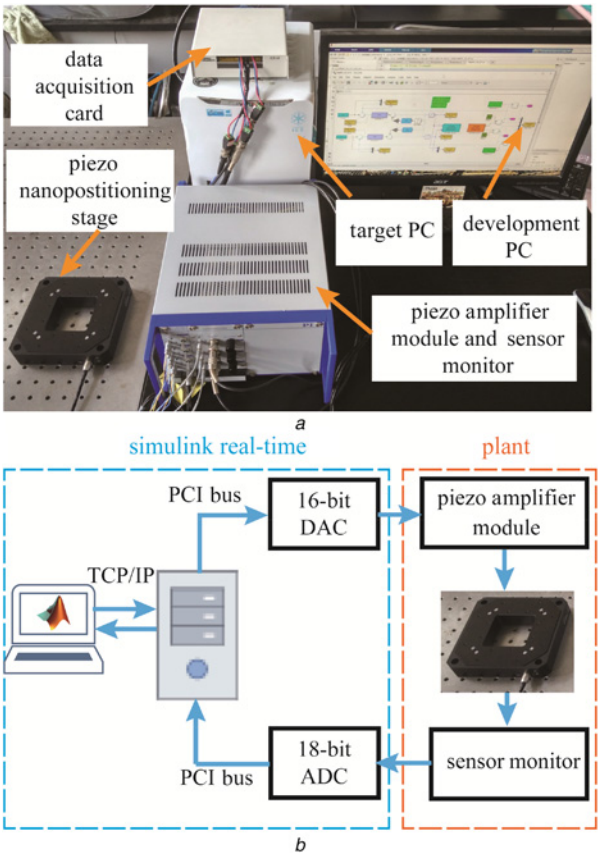

Fig. 1 *Experimental setup*
a Experimental platform
b Block diagram of control system

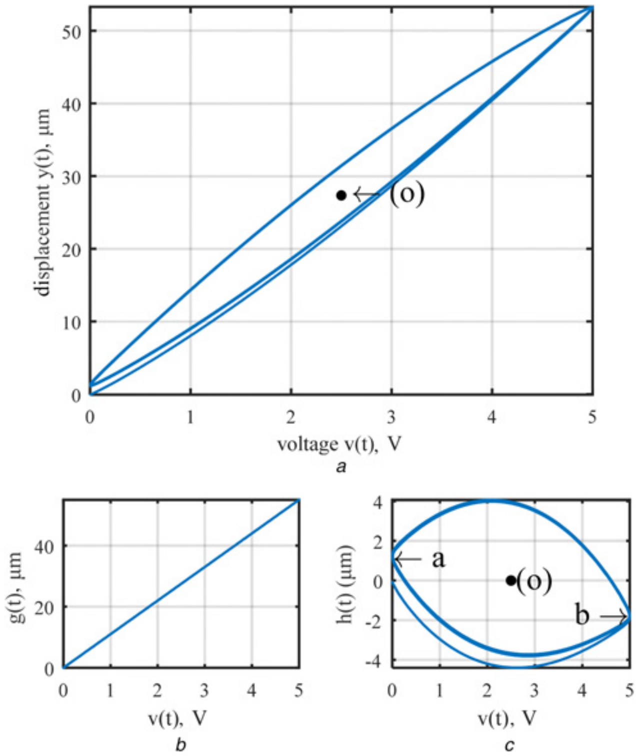

Fig. 2 *Experiment data of the PEA-actuated stage with the 0.1 Hz sine input voltage*
a Hysteresis loop
b Linear component
c Nonlinear component

phenomenon. Fig. 2*a* shows the measured hysteresis loop of the *x*-axis of the P561.3CD with a 0.1 Hz sinusoidal voltage input. Similar to that in [6], considering the fitted line in least-squares sense as the linear component, the hysteresis loop is decomposed into the linear component $g(t)$ and the nonlinear component $h(t)$, which are shown in Figs. 2*b* and *c*, respectively. It can be seen from Fig. 2*c* that the hysteresis nonlinear component is asymmetric about the centre point (o), namely the value 'a' is not equal the value '$-b$'. From the view of mathematics, the superposition of the two components can be used to describe the measured hysteresis curves.

The output displacement of PEA can be expressed by

$$\begin{cases} y(t) = g(t) + h(t) \\ g(t) = kv(t) \end{cases} \tag{1}$$

where $g(t)$ and $h(t)$ are both the functions of input voltage $v(t)$, k is a constant and $y(t)$ denotes the output displacement.

The CBW model has been extensively applied in piezoelectric hysteresis modelling, which takes on a nonlinear differential equation form with a few parameters. However, it can only describe symmetric hysteresis loop about the centre point. Its mathematical expression is as follows:

$$\dot{h}(t) = \alpha\dot{v}(t) - \beta|\dot{v}(t)||h(t)|^{n}h(t) - \gamma\dot{v}(t)|h(t)|^{n} \tag{2}$$

where $\dot{h}(t)$ is the derivative of $h(t)$, $v(t)$ and $\dot{v}(t)$ are, respectively, the applied input excitation and its derivative with respect to time, and α, β, γ, n are the model parameters. The coefficient α controls the amplitude of the hysteresis loop, while β, γ control the shape of the hysteresis loop and n controls the smoothness of the transition from elastic to plastic response [5]. Due to the properties and characteristics of the component material in the PEAs, $n = 1$ is generally utilised to characterise the hysteresis of the PEA-actuated nanopositioning stages [30].

Fig. 3 illustrates the simulated hysteresis loop generated by the CBW model $h(t)$ with randomly selected a set of parameters $\alpha = -0.08$, $\beta = 0.4$, $\gamma = -0.15$. It is obvious that the hysteresis loop generated by the CBW in Fig. 3 is indeed symmetric about the centre point (o), and the maximum value (A) and the minimum value (B) appear when the input signal is 0 and 5 V separately. However, Fig. 3 shows the actual nonlinear component is not only asymmetric about the centre point (o), but also related to the differential of the input, namely, the maximum value (C) and the minimum value (D) appear when the differential of the input is close to maximum and minimum instead of when the input signal is 0 and 5 V. Therefore, the modelling error is large when the CBW model is used to model the actual hysteresis component.

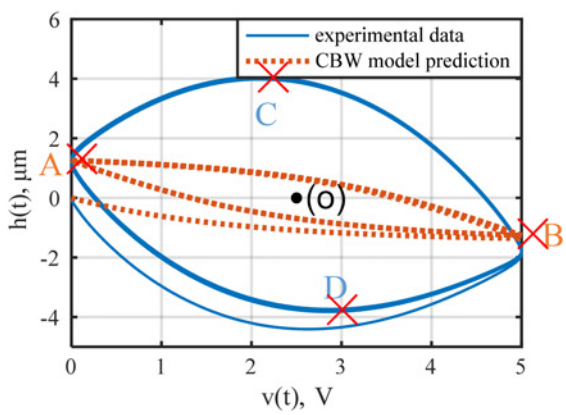

Fig. 3 *Nonlinear component (h(t)) predicted by the CBW model and the experiment data*

3. MBW model and parameters identification: To model the hysteresis of the PEA-actuated nanopositioning stages, this Letter adopts the asymmetric BW model in [6] for the asymmetric property, which introduces an asymmetric formula into the CBW hysteresis operator. Equation (2) can be rewritten as

$$\dot{h}(t) = \alpha\dot{v}(t) - \beta|\dot{v}(t)||h(t)|^{n-1}h(t) - \gamma\dot{v}(t)|h(t)|^{n} + \delta|\dot{v}(t)| \quad (3)$$

where δ is the asymmetric factor and $\delta\dot{v}(t)$ is the asymmetric formula. It has been verified that the asymmetric BW hysteresis model expressed as (3) can effectively characterise the asymmetric property of the hysteresis loop.

Besides, it should be mentioned that Fig. 2c shows the actual hysteresis component is also related to the differential of the input. The change trend of the nonlinear component is small-large-small with the input voltage from 0 to the max, which is similar to the differential of the input. An effective and simple approach is using the polynomial function of the differential of the input $\dot{v}(t)$ to characterise this property. The nonlinear component is afresh recorded as $w(t)$, defined as a generalised nonlinear input function as follows:

$$w(t) = h(t) + p\dot{v}(t) + q\dot{v}(t)^3 \quad (4)$$

Constructing a precise and effective model to describe the hysteresis of the PEA-actuated nanopositioning stages is one of the objectives of this work. The combination of (3) and (4) can define a MBW hysteresis model which is expressed as, where, k is a constant

$$\begin{cases} y(t) = g(t) + w(t) \\ g(t) = kv(t) \end{cases} \quad (5)$$

Different from the asymmetrical models in [6, 29, 30], the MBW model proposed in this work considers the asymmetry and the relation between the actual hysteresis component and the differential of the input simultaneously for modelling more accurately.

Several methods for identifying BW model parameters have been studied such as differential evolution algorithm [29], the nonlinear least squares method [30], the nonlinear filter system identification method [5] and PSO algorithm [26]. As a global method for solving both constrained and unconstrained optimisation problems, the PSO can be employed to solve a variety of optimisation problems [25]. Here, PSO is adopted to identify the modified BW model parameters k, α, β, γ, δ, p, q.

The fitness function is chosen as follows:

$$J_{\text{RMSE}} = \sqrt{\sum_{i=1}^{N}\left(y_{\exp}(i) - y_{\text{BW}}(i)\right)^2/N} \quad (6)$$

where $y_{\exp}(i)$ is the measured data from experiment at the ith sampling time, $y_{\text{BW}}(i)$ is the corresponding MBW model simulation output and N is the total number of samples. So the MBW model parameters optimisation problem can be described as

$$\min J_{\text{RMSE}} \quad \text{subject to} \quad \begin{cases} y = kv + p\dot{v} + q\dot{v}^3 + h \\ \dot{h} = \alpha\dot{v} - \beta|\dot{v}|h - \gamma\dot{v}|h| + \delta|\dot{v}| \end{cases} \quad (7)$$

The whole model identification process is carried out offline as follows:

(i) *Data collection*: With a full-range input sinusoidal voltage signal (0–5 V) in 0.1 Hz applied to the stage, the output displacement of the stage is measured and recorded.

(ii) *Model implementation*: The model is implemented with Matlab/Simulink as shown in Fig. 4. The model output is generated by the simulation.

(iii) *Model identification*: The PSO algorithm is performed for identifying the model parameters to match the simulation output to the experimental data.

4. Model-based feedforward compensation: The main goal of this Letter is to improve the positioning accuracy of the PEA-actuated nanopositioning stage by compensating the nonlinear component w. Therefore, the MBW model is used to estimate the hysteresis and a feedforward controller is used to compensate on this basis so that the relationship between the output displacement and the input control signal is linear.

Consider the desired displacement signal y_r applied to the controller. The reference input voltage v_r can be given by

$$v_r(t) = y_r(t)/k \quad (8)$$

According to (3) and (4), the estimated value $\hat{w}(t)$ of the nonlinear component $w(t)$ can be expressed as

$$\hat{w}(t) = \dot{\hat{h}}(t) + p\dot{v}_r(t) + q\dot{v}_r(t)^3 \quad (9)$$

$$\dot{\hat{h}}(t) = \alpha\dot{v}_r(t) - \beta|\dot{v}_r(t)||\hat{h}(t)|^{n-1}\hat{h}(t) - \gamma\dot{v}_r(t)|\hat{h}(t)|^{n} + \delta|\dot{v}_r(t)| \quad (10)$$

Consider the direct model in (5), the feedforward control v_{ff} can be given by

$$v_{ff}(t) = (y_r(t) - \hat{w}(t))/k = v_r(t) - \hat{w}(t)/k \quad (11)$$

The reference input voltage v_r instead of the actual input signal to the stage is used to calculate the estimated value $\hat{w}(t)$ of the nonlinear component with the merits of simplicity of implementation. However, there is a certain error between $\hat{w}(t)$ and $w(t)$. In this Letter, a proportional gain k_p is introduced to reduce the error, thus further improve the control accuracy. The value of k_p is determined by trial and error in the vicinity of unit value 1. So, the control voltage v_c can be descried as

$$v_c(t) = k_p v_r(t) - \hat{w}(t)/k \quad (12)$$

The block diagram of the model-based feedforward is shown in Fig. 5.

5. Experimental results: In this section, we identify the parameters of the three models: the CBW model, the asymmetrical BW model in [6] and the MBW model proposed in this Letter. The hysteresis curves shown in Fig. 6 are obtained by applying sine signals with a maximal amplitude of 5 V in different frequency from 0.02

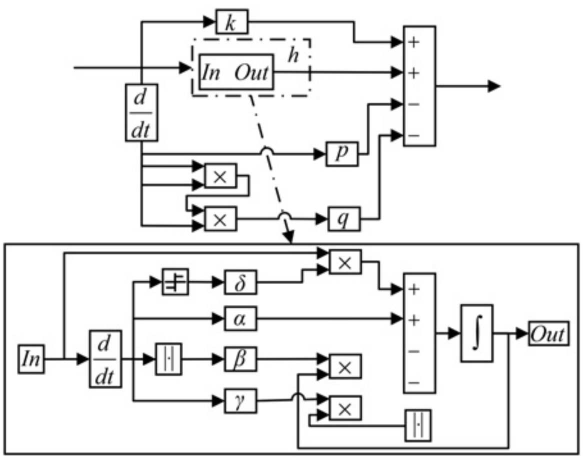

Fig. 4 *Block diagram of the MBW model*

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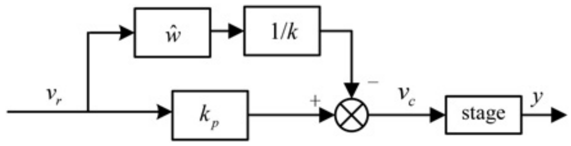

Fig. 5 *Hysteresis compensation structure of the model-based controller for the PEA-actuated stage*

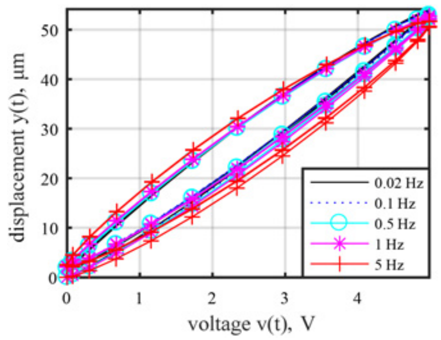

Fig. 6 *Hysteresis curves of the PEA-actuated stage under different input frequencies*

to 5 Hz. It can be seen that the shape of the hysteresis loop of the PEA-actuated nanopositioning stage changes little in the low-frequency voltage excitation from 0.02 to 1 Hz, i.e. the model is static. In order to avoid the effect of the creep on the hysteresis curve with a frequency lower than 0.02 Hz and the phase-lag due to the dynamics with a frequency higher than 1 Hz, we choose 0.1 Hz for the parameters identification and model validation.

The optimisation is carried out with a PSO toolbox running in Matlab environment, and the identified model parameters are shown in Table 1. With the obtained model, the comparison between the models output and experimental result is illustrated in Fig. 7.

It can be seen that the hysteresis curves generated by the obtained MBW model well match the experiment data of the PEA-actuated nanopositioning stage far better than the other two models. For the quantitative comparison, Table 2 shows the root mean square error (RMSE) and the relative error (RE) of the three different models.

$$\text{RMSE} = \sqrt{\sum_{i=1}^{N} (y_{\text{exp}}(i) - y_{\text{BW}}(i))^2 / N} \tag{13}$$

$$\text{RE} = \sqrt{\sum_{i=1}^{N} (y_{\text{exp}}(i) - y_{\text{BW}}(i))^2 \Big/ \sum_{i=1}^{N} (y_{\text{exp}}(i))^2} \tag{14}$$

Table 1 Identified parameters

Parameters	Model		
	CBW	Asymmetric BW	MBW
k	1.0974	1.0986	1.0998
α	−0.7051	−0.6715	−0.0656
β	1.9251	1.9231	0.3922
γ	−0.62	−0.35	−0.15
n	1	1	1
δ	—	0.032	−0.011
p	—	—	0.0108
q	—	—	3.68×10^{-5}

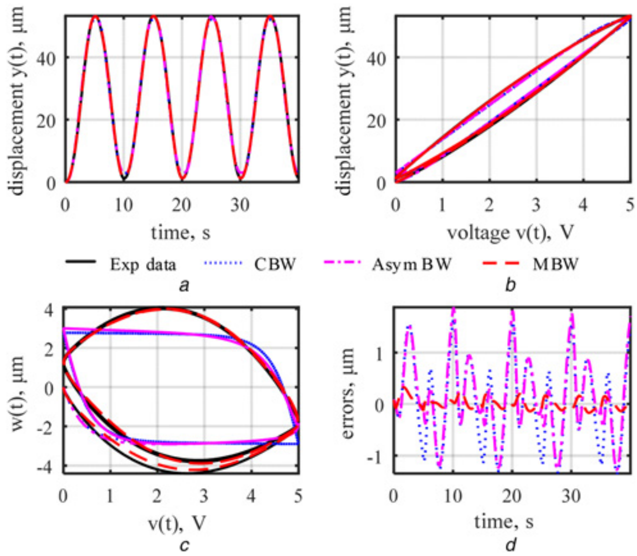

Fig. 7 *Comparison of the three different models*
a Displacement versus time
b Displacement versus input voltage
c Nonlinear component
d Prediction errors

Table 2 RMSE and RE of the three different models prediction

Model	CBW	Asymmetric BW	MBW
RMSE, μm	0.790	0.659	0.103
RE, %	2.38	1.94	0.31

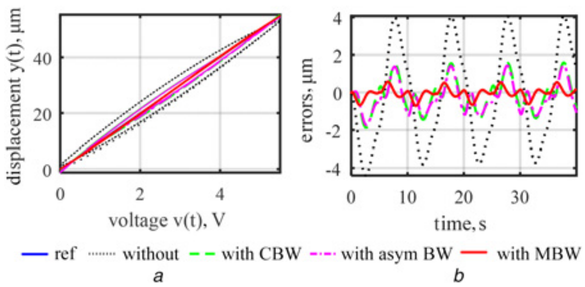

Fig. 8 *Experiment results with and without compensation under 0.1 Hz*
a Displacement versus input voltage
b Errors

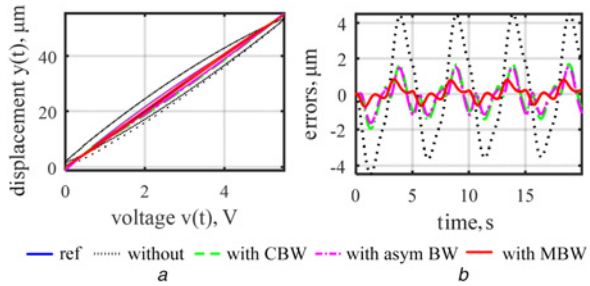

Fig. 9 *Experiment results with and without compensation under 0.2 Hz*
a Displacement versus input voltage
b Errors

Table 3 RMSE and RE of the displacement errors without compensation and with compensator based on the three models

Frequency	Control	Without	CBW	Asymmetric BW	MBW
0.1 Hz	RMSE, μm	2.613	0.837	0.808	0.286
	RE, %	4.75	1.52	1.47	0.52
0.2 Hz	RMSE, μm	2.728	0.867	0.781	0.373
	RE, %	4.96	1.58	1.42	0.68

The comparative experiment of hysteresis compensation is implemented in the established experimental system with the identified three models and the structure of the compensator as shown in Fig. 5.

The reference voltage v_r is a sine input reference with an amplitude 5 V and the frequency 0.1 Hz. Then the reference voltage v_r with the same amplitude 5 V and the frequency 0.2 Hz is tested. The experiment results of the control system are compared in Figs. 8 and 9. It can be seen that the performance of the compensator based on the MBW model is obviously better than the other two. The initial hysteresis has been almost completely removed with the feedforward compensator based on the MBW model. Table 3 shows the RMSE and RE of the displacement errors in Figs. 8 and 9. RE is obtained by RMSE dividing the total travel.

6. Conclusion: In this Letter, a novel modified BW model for modelling the hysteresis of the PEA-actuated nanopositioning stages is established by introducing a polynomial function of the differential of the input. The PSO algorithm is adopted to identify the model parameters, and the obtained hysteresis model can accurately match the actual experimental output of the stage with only 0.31% RE. In order to improve the position accuracy of stage, the model-based feedforward controller is proposed and developed with the advantage of no need for operator or model inversion. The experimental results show that the proposed approach is very useful for the improvement of the position accuracy of the stage. Feedback control and better estimation techniques are promising for improving the positioning accuracy of the PEA-actuated nanopositioning stage. We will conduct research on combining feedback and estimation techniques in our future work.

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8 References

[1] Kenton B.J., Leang K.K.: 'Design and control of a three-axis serial-kinematic high-bandwidth nanopositioner', *IEEE/ASME Trans. Mechatronics*, 2012, **17**, (2), pp. 356–369
[2] Aridogan U., Shan Y., Leang K.K.: 'Design and analysis of discrete-time repetitive control for scanning probe microscopes', *J. Dyn. Sys. Meas. Control*, 2009, **131**, (6), p. 061103
[3] Li Y., Xu Q.: 'Modeling and performance evaluation of a flexure-based XY parallel micromanipulator', *Mech. Syst. Signal Proc.*, 2009, **44**, (12), pp. 2127–2152
[4] Niezrecki C., Brei D., Balakrishnan S., ET AL.: 'Piezoelectric actuation: state of the art', *Shock Vib. Dig.*, 2001, **33**, (4), pp. 269–280
[5] Rakotondrabe M.: 'Bouc–Wen modeling and inverse multiplicative structure to compensate hysteresis nonlinearity in piezoelectric actuators', *IEEE Trans. Autom. Sci. Eng.*, 2011, **8**, (2), pp. 428–431
[6] Zhu W., Wang D.: 'Non-symmetrical Bouc–Wen model for piezoelectric ceramic actuators', *Sens. Actuator A-Phys.*, 2012, **181**, pp. 51–60
[7] Lin C.J., Lin P.T.: 'Tracking control of a biaxial piezo-actuated positioning stage using generalized Duhem model', *Comput. Math. Appl.*, 2012, **64**, (5), pp. 766–787
[8] Lin C.J., Yang S.R.: 'Precise positioning of piezo-actuated stages using hysteresis-observer based control', *Mechatronics*, 2006, **16**, (7), pp. 417–426
[9] Zhu W., Rui X.T.: 'Hysteresis modeling and displacement control of piezoelectric actuators with the frequency-dependent behavior using a generalized Bouc–Wen model', *Precis. Eng.*, 2016, **43**, pp. 299–307
[10] Wu Y., Zou Q.: 'Iterative control approach to compensate for both the hysteresis and the dynamics effects of piezo actuators', *IEEE Trans. Control Syst. Technol.*, 2007, **15**, (5), pp. 936–944
[11] Ling J., Feng Z., Yao D., ET AL.: 'A position domain iteration learning control for contour tracking with application to a multi-axis motion testbed'. IEEE American Control Conf. (ACC), Boston, MA, USA, 2016, pp. 1247–1252
[12] Feng Z., Ling J., Ming M., ET AL.: 'A model-data integrated iterative learning controller for flexible tracking with application to a piezo nanopositioner'. Transactions of the Institute of Measurement and Control, 0142331217719958, 2017
[13] Guo Y., Zhang Z., Zhou K., ET AL.: 'Robust control of rate-dependent hysteresis in smart actuators'. 2016 35th Chinese IEEE Control Conf. (CCC), Chengdu, China, 2016, pp. 837–841
[14] Zhang Z., Lu J., Zhou K., ET AL.: 'Dynamic hysteresis modeling and control of piezoelectric actuator based on H∞ robust disturbance observer'. 2016 35th Chinese. IEEE Control Conf. (CCC), Chengdu, China, 2016, pp. 897–902
[15] Xu Q.: 'Continuous integral terminal third-order sliding mode motion control for piezoelectric nanopositioning system', *IEEE-ASME Trans. Mechatron.*, 2017, **22**, (4), pp. 1828–1838
[16] Xu Q.: 'Precision motion control of piezoelectric nanopositioning stage with chattering-free adaptive sliding mode control', *IEEE Trans. Autom. Sci. Eng.*, 2017, **14**, (1), pp. 238–248
[17] Xu Q.: 'Digital integral terminal sliding mode predictive control of piezoelectric-driven motion system', *IEEE Trans. Ind. Electron.*, 2016, **63**, (6), pp. 3976–3984
[18] Gu G.Y., Zhu L.M., Su C.Y., ET AL.: 'Modeling and control of piezo-actuated nanopositioning stages: a survey', *IEEE Trans. Autom. Sci. Eng.*, 2016, **13**, (1), pp. 313–332
[19] Song G., Zhao J., Zhou X., ET AL.: 'Tracking control of a piezoceramic actuator with hysteresis compensation using inverse Preisach model', *IEEE-ASME Trans. Mechatron.*, 2005, **10**, (2), pp. 198–209
[20] Rosenbaum S., Ruderman M., Strohla T., ET AL.: 'Use of jiles–atherton and preisach hysteresis models for inverse feed-forward control', *IEEE Trans. Magn.*, 2010, **46**, (12), pp. 3984–3989
[21] Liaw H.C., Shirinzadeh B.: 'Robust adaptive constrained motion tracking control of piezo-actuated flexure-based mechanisms for micro/nano manipulation', *IEEE Trans. Ind. Electron.*, 2011, **58**, (4), pp. 1406–1415
[22] Gu G.Y., Zhu L.M., Su C.Y.: 'Modeling and compensation of asymmetric hysteresis nonlinearity for piezoceramic actuators with a modified Prandtl–Ishlinskii model', *IEEE Trans. Ind. Electron.*, 2014, **61**, (3), pp. 1583–1595
[23] Shan Y., Leang K.K.: 'Dual-stage repetitive control with prandtl–ishlinskii hysteresis inversion for piezo-based nanopositioning', *Mechatronics*, 2012, **22**, (3), pp. 271–281
[24] Esbrook A., Tan X., Khalil H.K.: 'Control of systems with hysteresis via servocompensation and its application to nanopositioning', *IEEE Trans. Control Syst. Technol.*, 2013, **21**, (3), pp. 725–738
[25] Xu Q., Li Y.: 'Dahl model-based hysteresis compensation and precise positioning control of an XY parallel micromanipulator with piezoelectric actuation', *J. Dyn. Sys. Meas. Contronl*, 2010, **132**, (4), p. 041011
[26] Li Y., Xu Q.: 'Adaptive sliding mode control with perturbation estimation and PID sliding surface for motion tracking of a piezo-driven micromanipulator', *IEEE Trans. Control Syst. Technol.*, 2010, **18**, (4), pp. 798–810
[27] Xu Q.: 'Identification and compensation of piezoelectric hysteresis without modeling hysteresis inverse', *IEEE Trans. Ind. Electron.*, 2013, **60**, (9), pp. 3927–3937
[28] Xu Q., Tan K.K.: 'Feedforward control without modeling inverse hysteresis' (Advanced Control of Piezoelectric Micro-/Nano-Positioning Systems. Springer International Publishing, Switzerland, 2016), pp. 57–75
[29] Wang G., Chen G., Bai F.: 'Modeling and identification of asymmetric Bouc–Wen hysteresis for piezoelectric actuator via a novel differential evolution algorithm', *Sens. Actuator A-Phys.*, 2015, **235**, pp. 105–118
[30] Gu G.Y., Zhu L.M.: 'Modeling of rate-dependent hysteresis in piezoelectric actuators using a family of ellipses', *Sens. Actuator A-Phys.*, 2011, **165**, (2), pp. 303–309

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