PID-type sliding mode-based adaptive motion control of a 2-DOF piezoelectric ultrasonic motor driven stage

Min Ming, Wenyu Liang, Zhao Feng, Jie Ling, Abdullah Al Mamun, Xiaohui Xiao

A B S T R A C T

This paper presents a new scheme of adaptive sliding mode control (ASMC) for a piezoelectric ultrasonic motor driven X–Y stage to meet the demand of precision motion tracking while addressing the problems of unknown nonlinear friction and model uncertainties. The system model with Coulomb friction and unilateral coupling effect is first investigated. Then the controller is designed with adaptive laws synthesized to obtain the unknown model parameters for handling parametric uncertainties and offsetting friction force. The robust control term acts as a high gain feedback control to make the output track the desired trajectory fast for guaranteed robust performance. Based on a PID-type sliding mode, the control scheme has a simple structure to be implemented and the control parameters can be easily tuned. Theoretical stability analysis of the proposed novel ASMC is accomplished using a Lyapunov framework. Furthermore, the proposed control scheme is applied to an X–Y stage and the results prove that the proposed control method is effective in achieving excellent tracking performance.

1. Introduction

There are many applications nowadays that require precision motion control, e.g., atomic force microscopy [1], nanopositioner [2,3] microgripper [4], robot-assisted surgeries, medical devices [5–7], and so on. In these applications, piezoelectric actuators are widely used because of their ultra-high resolution and fast response, which make them achieve high-precision and high-speed motion. Piezoelectric ultrasonic motor (PUM) is a type of direct-drive motor that is powered by a piezoelectric component [8]. Due to the merits of the piezoelectric component, the PUM has a compact size and can achieve high-speed and high-precision motion. However, designing a controller that can achieve high tracking performance is still a challenge to these motion systems due to nonlinear behavior of actuators, parametric uncertainties, and unmodeled disturbances. For a PUM, the nonlinear friction force is the main factor affecting tracking performance. Furthermore, a 2-DOF stage, consisting of two orthogonal ultrasonic motors, gives rise to problems due to coupling if (a) there is any deviation from angles at the time of installation of the stage, and (b) there is any misalignment between the center of the load and the center of the stage [9].

Many robust controllers have been proposed in the published literature to address the issues of uncertainties and disturbances, e.g., disturbance observer-based control [10–13], sliding mode control (SMC) [14–17], adaptive robust control (ARC) [18], intelligent control [8], neural network-based (NN-based) control [19,20], and so on. These works used different devices to underscore the effectiveness of their methods. In observer-based controllers, nonlinearities, model uncertainties, and disturbances are treated as a lump disturbance to a nominal linear model. A disturbance observer is often adopted to observe and compensate the lump disturbance and feedback control is used to achieve the object of desired tracking performance. SMC is a practical variable structure control to cope with all bounded disturbances and achieve asymptotic tracking performance. An extended state observer (ESO) combined with SMC is an effective strategy [21]. In [11], disturbance compensation via ESO is developed and a model-based second-order sliding function is designed. It also concludes that the integral sliding mode control can improve the tracking performance by introducing the integral term of tracking error [13]. However, ESO is limited by its bandwidth and, therefore, can only perform well for low-frequency reference trajectories. It is reported in [22] that a static friction model can predict the friction phenomenon with almost the same performance as using a dynamic friction model. The static friction model is identified and used for feedforward compensation in [22,23]. Although many friction models are proposed in [24], the traditional compensation schemes using the static friction model may not be valid in some...
practical applications where the friction force is varying and uncertain. Therefore, unknown model uncertainties and friction force should be estimated and compensated in real-time to improve the control accuracy of SMC and achieve tracking faster desired trajectories.

Adaptive control is a good choice for improving tracking accuracy when model parameters to be estimated are unknown but constant. A robust controller that merges sliding-mode and adaptive schemes is proposed in [25], where the adaptive part is used to cope with unknown slow-time varying disturbances. ARC proposed in [26] is very suitable for a class of systems with unknown non-linear model. It combines parametric adaptive law with deterministic robust control, namely high gain feedback control. The system model established in [26] consists of a dynamic part with Coulomb friction, and the unknown parameters are estimated using adaptive law. This control scheme has been successfully applied to many practical applications [18, 27, 28]. However, it is mentioned in [29] that ARC can only guarantee the tracking error convergences to a specified limit set because the feedback control gains cannot be set to infinity. The robust integral of the sign of error (RISE) controller developed in [30] can achieve asymptotic tracking performance when the disturbance is assumed to be second-order derivable and bounded [31]. A RISE-based adaptive controller (ARISE) is proposed in [29] for a hydraulic system. It is worth noting that for a second-order system, the essential difference between ARISE and ARC is that the high gain feedback control law in ARISE includes the integral terms of the error and the sign of the error. When the disturbance is small, the integral term of the sign of the error is negligible. It is also verified that introducing integral terms of error to the control law can improve the tracking performance in [13, 32]. It should be noted that an integral sliding surface not only can reduce the steady-state error but also can provide faster response and more accurate trajectory tracking performance than the common PD-type sliding surface. To integrate the integral term, the PID-type sliding mode function has been adopted in [32–34].

Though the existing control strategies for systems with nonlinear friction and uncertainty show great improvements, the following two aspects can be further improved: (a) it is important to adopt active compensation to alleviate the non-linearity and uncertainty so that the controller can achieve improved performance and be suitable for fast tracking, and (b) the asymptotic tracking performance can be obtained by integrating the benefits of the SMC and the advanced controller ARISE, which can be realized by using a PID-type sliding mode function. Therefore, this paper presents a novel adaptive sliding mode control (ASMC) by combining PID-type sliding surface-based SMC and adaptive control, which makes the proposed controller have advantages in terms of strong robustness and high tracking performance.

The main contributions of this paper can be summarized as follows. First, it is desirable that a controller is designed with a clear structure, is easy in tuning, and can achieve high tracking performance. There is no requirement to have a specific knowledge of the friction and cross effect. The adaptive control law is designed to estimate the model parameters and disturbance simultaneously. The adaptive control is designed to realize active compensation and relieve the chattering phenomenon. The PID-type sliding mode including the integral term with a simple structure can further improve the tracking performance. More precisely, the proposed controller (ASMC) consists of two terms: (1) a model compensation term related to the reference trajectory which is adjustable with estimating model parameters via adaptive technique, and (2) a feedback robust term. The stability of the proposed control scheme is proven in the Lyapunov framework. Finally, to demonstrate the effectiveness of the proposed controller in achieving high tracking performance, extensive comparative experiments are conducted on an ultrasonic-motor-driven 2-DOF stage.

The main content of this paper is organized as follows. The system description and control problem statement are introduced in Section 2. Section 3 gives the detailed steps of controller design and stability analysis, and the experimental setup is described in Section 4. Experiments and comparisons of the results are presented and discussed in Section 5. Some conclusions are summarized in Section 6.

2. System description and control problem statement

2.1. 2-DOF system

The 2-DOF piezoelectric ultrasonic motor (PUM) driven stage consists of two PUMs, shown in Fig. 1. The PUM moving along the Y-axis is orthogonally mounted on the PUM moving along the X-axis, i.e., the stator (base) of Y-axis PUM is connected to the mover of X-axis PUM via threaded fasteners. Due to the series connection between the two PUMs, there is no motion along Y-axis when only the X-axis PUM is driven, while there is motion along X-axis because of interacting friction force when only the Y-axis PUM is driven. The coupling effect exists in this X-Y stage.

For each PUM, the piezoelectric component induced in the stator is used both to impart motion and to modulate the frictional forces present at the interface, and the output platform is bonded on the mover moving along the linear guide. The ultrasonic vibration of the piezoelectric component can power the motion of the moving end. During the motion, the friction at the interface between the moving end and the corresponding unmoving base arises and results the system affected by nonlinearity.

2.1.1. Y-axis

The Y-axis PUM acts as a single-axis PUM. In [35], the hysteresis, creep, cross-coupling, and external disturbance are consolidated into an output disturbance. Similar to the modeling approach in [35], the unmodeled uncertainty and disturbance are treated as a total disturbance. The following dynamic model is used to describe it [23]:

\[ y(t) + a_{y1}y(t) + a_{y0}y(t) = b_{y0}u(t) + f_y(t) + d_y(t), \]

where \( u \) and \( y \) are the control input and position output, respectively. \( a_{y1}, a_{y0}, \) and \( b_{y0} \) are the model coefficients. \( f_y \) is the nonlinear term and \( d_y \) is disturbance.

The nonlinear term is resulted from the characteristics of the piezoelectric component and the drive mechanism, including hysteresis and friction. Frictional force at the interface is the major nonlinear component of this PUM. In this paper, the nonlinear term is described by a Coulomb friction model \( f_{\mu} \) added to an uncertain nonlinear component \( \Delta f_y \).

\[ f_y(t) = f_{\mu}(t) + \Delta f_y(t). \]

It is found from the open loop test that, the friction is asymmetric in forward and backward motions [23]. Thus the model of the nonlinear term can be modeled by Coulomb friction plus a constant,

\[ f_{\mu}(t) = f_{\mu} \text{sign}(t) + f_{I2}, \]

where ‘sign’ is the signum function, and \( f_{\mu}, f_{I2} \) are unknown model coefficients.
In practical situations, the bounded model uncertainty and disturbance are combined into $p_i(t)$, 
\[
p_i(t) = DF_i(t) + D_i(t),
\]
(4) 

2.1.2. X-axis

Due to the physical configuration of the stage, the dynamics of the X-axis PUM is affected by the coupling with Y-axis motion. Considering the motion principle of PUM, the friction force drives the moving end, X-axis motion is also partly affected by the friction force of Y-axis when the two axes are non-orthogonal and interact with each other. 

The dynamics model of the X-axis PUM can be expressed as:
\[
\ddot{x}_{\text{1}}(t) = a_{\text{1}}\ddot{x}_{\text{1}}(t) + a_{\text{2}}x_{\text{1}}(t) = b_{\text{1}}u_{\text{1}}(t) + f_{\text{1}}(t) + D_{\text{1}}(t),
\]
(5) 

where $f_{\text{1}}$ is the coupling from Y-axis motion to X-axis, $x$ is the output displacement, and other corresponding parameters have the same definitions like those for Y-axis.
\[
f_{\text{1}}(t) = f_{\text{1}}(t) + DF_{\text{1}}(t),
\]
(6) 

\[
f_{\text{1}}(t) = f_{\text{1}}(t) + DF_{\text{1}}(t),
\]
(7) 

where $f_{\text{1}}$, $f_{\text{2}}$, $c_1$, $c_2$ are unknown model coefficients.

\[
\dot{p}_i(t) = DF_i(t) + D_i(t),
\]
(9) 

2.2. Control problem statement

The system model can be expressed in a matrix form, 
\[
\begin{align*}
\dot{q} + B\dot{q} + Kq &= Tu + FS(q) + d_i + d, \\
\end{align*}
\]
(12) 

for conciseness, where $q = [y(t), x(t)]^T$, $\dot{q} = [\dot{y}(t), \dot{x}(t)]^T$, and $\ddot{q} = [\ddot{y}(t), \dddot{x}(t)]^T$ are vectors of the axis position, velocity, and acceleration, respectively. $u = [u_1(t), u_2(t)]^T$ is the control input, $S(q) = [\text{sign}(y), \text{sign}(x)]$ is a sign function of the velocity, $d_i = [d_{c1}, d_{c2}]^T$ is the nominal value of the unknown nonlinearity, and $d = [p_i(t), p_i(t)]^T$ is the residual disturbance. The control $d$ is assumed to be bounded 
\[
|d| \leq d_{\text{q}}, B = \text{diag}[a_{\text{1}}, a_{\text{2}}], K = \text{diag}[a_{\text{0}}, a_{\text{0}}], T = \text{diag}[b_{\text{1}}, b_{\text{0}}],
\]
and $F = [f_{\text{1}}, f_{\text{2}}]$ are matrices of model coefficients, where $\text{diag}(a)$ is a function that convert a vector to a diagonal matrix.

The control objectives are: (1) to achieve precise tracking of the reference trajectory $y_{ref}(t)$ for Y-axis by handling the adverse effects nonlinear frictional force $f_{\text{fric}}(t)$ and disturbance $p_i(t); (2)$ to achieve a precise tracking of $x_{\text{1}}(t)$ for X-axis in the presence of nonlinear frictional force $f_{\text{fric}}(t)$, unilateral coupling $f_{\text{1}}(t)$ and disturbance $p_i(t).$ Design of an ASMC control scheme based on PID-type sliding mode to achieve these control objectives is presented in the next section.

3. Controller design

To overcome these control issues, an adaptive robust controller is designed. Let us define the position error, 
\[
e(t) = q(t) - q_{\text{ref}}(t),
\]
(13) 

where $q_{\text{ref}}(t) = [y_{\text{ref}}, x_{\text{ref}}]^T$ is the desired position trajectories.

A PID type of sliding mode function based on the error (13) can be designed as:
\[
\sigma(t) = [\sigma_1, \sigma_2]^T = \dot{e}(t) + k_1e(t) + k_2\int_0^t e(\tau)d\tau,
\]
(14) 

where $k_1 = \text{diag}[k_{\text{11}}, k_{\text{12}}]$ and $k_2 = \text{diag}[k_{\text{21}}, k_{\text{22}}]$ are control gain matrices. It is easy to tune the control gains according to the rule that the characteristic polynomial $s^2 + k_{\text{11}}s + k_{\text{12}} = 0$ and $s^2 + k_{\text{21}}s + k_{\text{22}} = 0$ need to be strictly Hurwitz, where $s$ is the Laplace operator, i.e., the roots of the polynomials are with negative real parts. The gains are selected as positive values in the subsequent experiments.

**Theorem 1.** For the system synthesized by (12), if the controller (15) is adopted, the system can maintain robustness and enable the position tracking error converging to zero asymptotically.
\[
u = T^{-1}[\ddot{q}_y + \dot{K}q - \dot{F}S(q) - \ddot{d}_a - k_1e - k_2e - k_s\sigma - \beta\text{sign(}\sigma\text{)}],
\]
(15) 

where $\dddot{\nu}$ is the estimated value of $\dddot{\nu}$, and $k_s = \text{diag}[k_{s1}, k_{s2}]$, and $\beta = \text{diag}(\beta)$ with $B = [\beta_1, \beta_0]^T$ are positive control parameters that are designed to be $B \geq \text{diag}(H)$, where $H = [\eta, \eta]^T$ is a vector contains the diagonal elements of a designed positive diagonal matrix $\eta$ that can be arbitrarily small, i.e., $\eta = \text{diag}(H)$. $u_1$ consists of the desired trajectory and the model-based adaptive compensation, where the desired trajectory is usually known in advance.

**Proof of Theorem 1.** The model based adaptive control term is presented in detail, 
\[
u_a = \dot{q}_a + \dot{K}q - \dot{F}S(q) - \ddot{d}_a = [u_{\text{1a}}, u_{\text{2a}}]^T.
\]
(16) 

For clearer presentation, the corresponding control laws $u_{\text{1a}}, u_{\text{2a}}$ for X-axis and $u_a, u_b$ for Y-axis are expanded as follows.
\[
u_a = \dot{q}_a + \dot{K}q - \dot{F}S(q) - \ddot{d}_a = \begin{bmatrix} u_{\text{1a}}, u_{\text{2a}} \end{bmatrix}^T
\]
(17) 

where $\dddot{\nu}_a$ denotes the estimated value of $\dddot{\nu}_a$ and $\dddot{\theta}_a$ is the estimation error ($\dddot{\theta}_a = \dddot{\theta}_a - \dddot{\theta}_a$) and $\Gamma_a$ is the diagonal adaptation rate matrix.

Similar control design process is used for the X-axis, in which the coupling is taken into account.
\[
u_a = \dot{q}_a + \dot{K}q - \dot{F}S(q) - \ddot{d}_a = \begin{bmatrix} u_{\text{1a}}, u_{\text{2a}} \end{bmatrix}^T
\]
(18) 

where $\dddot{\nu}_a$ denotes the estimated value of $\dddot{\nu}_a$ and $\dddot{\theta}_a = \dddot{\theta}_a - \dddot{\theta}_a$ and $\Gamma_a$ is the diagonal adaptation rate matrix.

To facilitate the subsequent stability analysis of the system with the control, a Lyapunov function $V(t)$ is chosen as
\[
V(t) = \frac{1}{2}\dot{e}^T + \frac{1}{2}\dot{\sigma}^T + \frac{1}{2}\dot{\theta}_a^T + \frac{1}{2}\dot{\theta}_a^T + \frac{1}{2}\dot{\theta}_a^T + \frac{1}{2}\dot{\theta}_a^T
\]
(21) 

With the system model (12) and the control law (15), it can be obtained,
\[
\sigma = \dot{e} + k_1e + k_2e
\]
(22) 

Taking the derivative of (21), and taking into account (22), the adaptive rules (18) and (20), the resulting expression can be simplified.
\[
V = \sigma^T \sigma + \theta^T \Gamma^{-1} \dot{\theta} + \theta^T \Gamma^{-1} \dot{\theta}
\]
\[
= \sigma^T [Bq + Kq - FS(q) - d_n + d - k, \sigma - \beta \text{sign}(\sigma)]
\]
\[
- \theta^T \phi_1^T \phi_1 \dot{\theta}
\]
\[
= \sigma^T [Bq + Kq - FS(q) - d_n + d - k, \sigma - \beta \text{sign}(\sigma)]
\]
\[
- \sigma^T [\theta^T, \phi_1^T, \phi_1^T]^T
\]
\[
= \sigma^T (d - k, \sigma - \beta \text{sign}(\sigma)).
\]

Remark 3. If \( B \geq |d| \), it can be seen that the system is stable as long as \( k_i \geq 0 \), \( k_i \) is the key to affect the rate of reaching the sliding mode surface. However, \( \beta \text{sign}(\sigma) \) is a discontinuous term, large value of \( \beta \) may cause chattering problem. To avoid this issue, \( \beta \) needs to be set as small as possible. Taking into account the stability and rapidity of convergence, \( \beta \) takes a smaller value and \( k_i \) takes a larger value so that the controller can achieve high tracking performance.

4. Experimental setup

4.1. Experimental system

Fig. 3 shows the experimental setup. The system consists of two PUMs (model: C-185, from Physik Instrumente Co., Ltd.) to realize X and Y direction motion. Each PUM has a stroke limit of ±10 mm, and the integrated linear encoder used for output displacement measurement has a resolution of 0.1 \( \mu \)m. A dSPACE DS1104 embedded rapid prototyping system is employed to implement the controller with a sampling frequency of 1 kHz. A control PC is used to realize real-time interface and control.

4.2. System identification

In order to identify the nominal model of the stage, an input signal containing multiple frequency components is applied. The input-output data is collected and imported to MATLAB for identification. Interested readers may refer to an earlier publication [8] for details. Identified transfer functions are:

\[
G_y = \frac{2299}{s^2 + 182.7s + 460.8}
\]

and

\[
G_x = \frac{4940}{s^2 + 202s + 248.4}
\]

The corresponding coefficients \( b_0 = 2299 \), \( a_{11} = 182.7 \), \( a_0 = 460.8 \) and \( b_0 = 4940 \), \( a_{11} = 202 \), \( a_0 = 248.4 \) will be used in the controllers.
To conduct a comparison, three different controllers are designed and implemented: (1) proposed ASMC, (2) conventional PID control, and (3) SMC based on PID-sliding mode assisted by ESO.

ASMC: This is the adaptive SMC proposed in this paper. The controller parameters are $k_1 = k_2 = k_3 = \text{diag}(w_{e_x}, w_{e_y})$ with $w_{e_x} = 2\pi \times 80, w_{e_y} = 2\pi \times 100$ and $\beta = \text{diag}(0.001, 0.001)$ and the adaptive rates are $\Gamma_x = \text{diag}(100 \ 1000 \ 10), \text{and} \ \Gamma_y = \text{diag}(10 \ 1000 \ 10)$. The initial values of the adaptive model parameters are set as the identified values. The initial values of the nonlinear term coefficients and disturbances are set to zero.

It should be noted that the coupling effect in the 2-DOF stage is unilateral coupling, not cross coupling. The designed control is not cross-coupled control, so there is no control allocation problem in the experiments.

PID: Proportional–Integral–Derivative controller is widely used in the industry. To ensure that the feedback gains are identical with the proposed controller, the PID parameters are set as $k_1 = k_2 = k_3 = \text{diag}(w_{e_x}, w_{e_y})$ and $k_4 = \text{diag}(2w_{e_y}, 2w_{e_x})$.

$u_{PID} = -\text{T}^{-1}(k\dot{e} + k_s e + k_d \int e)$.

where $\text{T} = \text{diag}(b_{1o}, \ b_{2o})$.

SMC-ESO: The sliding mode control (SMC) with extended state observer (ESO) is selected as an advanced comparative controller. It has been successfully used in many applications as previously mentioned [11, 12, 21]. This controller also has advantage in dealing with model uncertainties and disturbances. The controller is redesigned in this paper, based on the same PID-type sliding mode. The unmodeled nonlinear term and disturbance is treated as an extra state of the system, and ESO is used to estimate and compensate it.

To verify the coupling compensation effect of the proposed controller, a sine wave $y_2 = \sin(2\pi x + 10\pi)$ is used for the motion along the $Y$-axis while keeping the $X$-axis with zero input. The $X$-axis position outputs in open-loop and closed-loop are shown in Figs. 4(b) and 4(c) respectively. It can be seen that in closed-loop the $X$-axis with the proposed controller can effectively suppress the coupling error from $Y$-axis to $X$-axis. Then a circular motion is conducted, the reference trajectories are $y_2 = \sin(2\pi x + 10\pi)$ and $x_2 = 1 - \cos(2\pi x + 10\pi)$. In case 1, the controller for $X$-axis is implemented without coupling adaption (i.e., $f_{c1} = 0$), while in case 2 it is implemented with coupling adaption.

The tracking errors in the two cases are shown in Fig. 5. The RMS errors of $X$-axis are 0.0074 mm in case 1 and 0.0068 mm in case 2. It can be seen that considering coupling effect during design helps to improve the tracking accuracy of $X$-axis by 8.1%. The improvement is not obvious, which can be explained in two aspects: (a) the coupling effect is small compared with the control input; (b) the controller designed with high gain feedback control term can address such coupling as effectively.

5.2. Sine wave trajectory (circular motion)

In this experiment, the 2-DOF stage is controlled to track several circular motions. The performance indexes are collected in Table 1. It is easily observed, by comparing the results of PID and SMC-ESO, that ESO gives significant improvement in tracking accuracy by compensating the disturbance using the additional estimated state. However, with increasing frequency of circular motion, the tracking performance with SMCESO becomes worse because of the bandwidth limitation. When the reference changes fast, the ESO cannot estimate the lump disturbance accurately enough. On the contrary, the ASMC can hold high accuracy in fast changing reference tracking. Among the three controllers, ASMC has the best performance, the RMS errors of the 1–10 Hz reference signals are within 10 μm. In ASMC, the PID-type sliding mode provides a robust feedback control. On the basis of this, adaptive control is combined to improve the accuracy by using adaptation of the unknown model parameters to compensate nonlinearities and

\[
\begin{align*}
\mu_c &= \sqrt{\frac{1}{N} \sum_{i=1}^{N} e_i^2}, \\
M_e &= \max(|e|), \\
A_e &= \frac{1}{N} \sum_{i=1}^{N} \sqrt{e_i^2 + c_i^2},
\end{align*}
\]

where $N$ is the number of data samples used.

5.1. Coupling compensation

Comparison of performances of three different controllers are carried out through several experiments. For comparison, we consider the root-mean-square (RMS) error $\mu_c$, the maximum tracking error $M_e$, and the average composition error $A_e$, which are defined as,

\[
\begin{align*}
\mu_c &= \sqrt{\frac{1}{N} \sum_{i=1}^{N} e_i^2}, \\
M_e &= \max(|e|), \\
A_e &= \frac{1}{N} \sum_{i=1}^{N} \sqrt{e_i^2 + c_i^2},
\end{align*}
\]
uncertainties. The experimental results show that adaptive control is suitable for a class of systems with known structure but uncertain model parameters.

The experimental results of the circular trajectory (1 mm and 10 Hz) are shown in Figs. 6 and 7 for clear comparison. The proposed controller achieves better performance than the other two controllers in terms of transient and tracking accuracy. It is observed that the adaptive control can compensate nonlinear friction and model uncertainties effectively to achieve the best tracking performance with the same high gain feedback control. The circular motion is shown in Fig. 8.

5.3. Triangular wave trajectory

Besides, the experiment results of triangle waves have been conducted. Due to triangular waves containing high frequency harmonics, the reference with the frequency of 1 Hz and 2 Hz has been used in the experiments. The value of RMSE and MAXE tracking errors are listed in Table 2. The compared tracking results of triangle wave (2 Hz, 2 mm) are shown the Fig. 9.

<table>
<thead>
<tr>
<th>Freq (Hz)</th>
<th>Errors (μm)</th>
<th>PID</th>
<th>SMC-ESO</th>
<th>ASMC</th>
<th>PID</th>
<th>SMC-ESO</th>
<th>ASMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>μx</td>
<td>10.85</td>
<td>10.37</td>
<td>6.86</td>
<td>6.89</td>
<td>2.59</td>
<td>2.84</td>
</tr>
<tr>
<td></td>
<td>Mx</td>
<td>27.70</td>
<td>28.67</td>
<td>11.00</td>
<td>11.85</td>
<td>6.20</td>
<td>7.99</td>
</tr>
<tr>
<td></td>
<td>Ay</td>
<td>13.69</td>
<td>9.68</td>
<td>3.64</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>μx</td>
<td>31.52</td>
<td>30.99</td>
<td>20.17</td>
<td>23.27</td>
<td>5.51</td>
<td>5.69</td>
</tr>
<tr>
<td></td>
<td>Mx</td>
<td>47.83</td>
<td>48.58</td>
<td>30.65</td>
<td>30.45</td>
<td>15.22</td>
<td>13.19</td>
</tr>
<tr>
<td></td>
<td>Ay</td>
<td>43.78</td>
<td>30.16</td>
<td>7.26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>μx</td>
<td>45.96</td>
<td>46.50</td>
<td>31.53</td>
<td>33.49</td>
<td>6.76</td>
<td>7.68</td>
</tr>
<tr>
<td></td>
<td>Mx</td>
<td>67.21</td>
<td>70.57</td>
<td>42.72</td>
<td>45.01</td>
<td>17.42</td>
<td>18.32</td>
</tr>
<tr>
<td></td>
<td>Ay</td>
<td>65.12</td>
<td>45.57</td>
<td>9.23</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.4. Robustness test

To verify the proposed controller can deal with disturbance robustly, the extreme disturbance \( d_{\text{dist}} = 0.1 + 0.01 \sin(2\pi t - \pi/6) + 0.01 \sin(10\pi t) \) is randomly selected and applied to X-axis in the experiments. Fig. 10 shows the compared tracking results for 10 Hz sine wave.
reference trajectory, and the error bars shown in Fig. 11 compares the tracking performance of the controller with and without the extreme disturbance. It can be noted that the proposed method can reduce the impact of interference and achieve satisfied tracking accuracy. Table 3 gives the values of performance indexes.

5.5. Specified motion

Furthermore, to investigate the performance of the proposed controller in practical application, a specified motion trajectory (for an ear surgical operation) is chosen as an example, like that in [8]. Figs. 12 and 13 show the tracking results of the three comparative controllers. The performance indexes are collected in Table 4. For the tracking performance of ASMC, $\mu_x$ for X-axis and Y-axis are 3.41 $\mu$m and 3.68 $\mu$m, respectively. As seen in these data and figures, the proposed ASMC is valid for practical application and performs the best among the three controllers for the specified motion. (See Fig. 14.)

6. Conclusion

In this paper, a nonlinear adaptive robust controller with PID-type sliding mode is developed to deal with nonlinear friction and parametric uncertainties and coupling effects for a 2-DOF stage driven...
Fig. 10. Tracking results comparison of X-axis with the extreme disturbance.

Fig. 11. Tracking errors comparison of with and without the extreme disturbance.

by linear PUMs. The model of the stage is derived with consideration of all uncertainties. Then, with a defined PID-type sliding mode function, the adjustable model-based control is designed. The high gain feedback term makes the system stable while the adaptive control term compensates for nonlinearities and uncertainties to improve the tracking performance. The control strategy is analyzed using a Lyapunov function for theoretical proof of its stability. The proposed control scheme is easy to implement with the easy tuning of controller parameters. Experiments are conducted on a 2-DOF stage to demonstrate the effectiveness of the proposed control scheme, which shows the proposed ASMC control scheme performs better than the other two methods.

CRediT authorship contribution statement

Min Ming: Methodology, Data curation, Writing - original draft. Wenyu Liang: Resources, Writing - review & editing. Zhao Feng: Software. Jie Ling: Investigation, Conceptualization. Abdullah Al Mum: Supervision, Writing - review & editing. Xiaohui Xiao: Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work was supported by the China Postdoctoral Science Foundation under Grant No. 2018M642905, China Scholarship Council (CSC)
Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.mechatronics.2021.102543.

References


Min Ming received the B.S. degrees in Mechanical Engineering from School of Power and Mechanical Engineering, Wuhan University, Wuhan, China in 2016. She is currently pursuing the Ph.D. degree in Mechanical Engineering at Wuhan University, Wuhan, China. She is now as a visiting Ph.D. student at the Department of Electrical and Computer Engineering, National University of Singapore. Her research interest covers hysteresis compensation, precision motion control, and nano-positioner.

Wenyu Liang received the B.Eng. and M.Eng. degrees in mechanical engineering from the China Agricultural University, Beijing, China, in 2008 and 2010, respectively, and the Ph.D. degree in electrical and computer engineering from the National University of Singapore, Singapore, in 2014. He is currently a Scientist with the Institute for Infocomm Research, A*STAR, Singapore and also an Adjunct Assistant Professor with the Department of Electrical and Computer Engineering, National University of Singapore. His research interests mainly include robotics, mechatronics and automation, precision motion control and force control with applications in medical and industrial technology.

Zhao Feng received the B.S. degrees in Mechanical Engineering from School of Power and Mechanical Engineering, Wuhan University, Wuhan, China in 2014. From 2019 to 2020, he was as a visiting Ph.D. student at the Department of Electrical and Computer Engineering, National University of Singapore. He is currently pursuing the Ph.D. degree in Mechanical Engineering at Wuhan University, Wuhan, China. His research interests include vibration control, iterative learning control, nanopositioning and robotics.

Jie Ling received his B.S. and Ph.D. degrees in Mechanical Engineering from School of Power and Mechanical Engineering, Wuhan University, China, in 2012 and 2018, respectively. He was a joint Ph.D. student with Department of Automatic Control and Micro-Mechatronic Systems, FEMTO-st Institute, France in 2017. From 2019 to 2020, he is a joint Postdoc Researcher with Department of Biomedical Engineering, National University of Singapore, Singapore. Since 2018, he has been a Postdoctoral Researcher with Department of Mechanical Engineering, Wuhan University, Wuhan, China. His research interests include mechanical design and precision motion control of nanopositioning stages and micromanipulation robots.

Abdullah Al Mamun is an associate professor in the department of Electrical and Computer Engineering of the National University of Singapore. He obtained Bachelor of Technology (Honours) degree from Indian Institute of Technology (IIT), Kharagpur, and Ph.D. from National University of Singapore. His research interests include mechatronics, precision control systems, and intelligent control of motors and drives. He is a senior member of IEEE.

Xiaohui Xiao received the B.S. and M.S. degrees in Mechanical Engineering from Wuhan University, Wuhan, China, in 1991 and 1998, respectively, and the Ph.D. degree in mechanical engineering from Huazhong University of Science and Technology, Wuhan, China, in 2005. She joined the Wuhan University, Wuhan, China, in 1998, where she is currently a Full Professor with the Mechanical Engineering Department, School of Power and Mechanical Engineering. Her current research interests include mobile robotics, high-precision positioning control, and signal processing.