



Signal-Transformation-Based Repetitive Control of Spiral Trajectory for Piezoelectric Nanopositioning Stages

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I. INTRODUCTION

Abstract—In this article, a novel signal-transformation-based repetitive controller (STBRC) dedicated to the accurate and fast tracking of spiral trajectory with a time-variant amplitude is proposed and tested. To overcome the requirement of strictly periodic references of conventional repetitive controller, a mapping between spiral trajectory and constant amplitude sinusoid is developed to provide multiple lower gains at the fundamental frequency and its harmonics of the spiral trajectory to ensure zero steady-state errors. The stability of the control method and convergence of steady-state errors are analyzed with a lifted-system representation, respectively, for its time-variant property. Compared with internal model-based controllers for this trajectory, STBRC has the advantages of a simpler structure, fewer parameters, and better performance. The application to a piezoelectric nanopositioning stage verifies the effectiveness of the proposed control method on suppression of cross-coupling errors and improving the tracking performance of high-speed spiral trajectory in comparison with other controllers.

Index Terms—Piezoelectric nanopositioning stage, repetitive control, signal transformation approach, spiral scan.

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THE piezoelectric nanopositioning stage is a crucial scanning device aimed at operating with nanometer or sub-nanometer precision. It has the advantages of high stiffness and fast response through employing piezoelectric actuators and flexure-hinge-guided mechanisms [1], especially in the application of scanning probe microscopes (SPMs) manipulating matter at the molecular and atomic levels [2]. In general, the raster-based trajectory has been widely employed to construct the scanning image in the SPMs. The raster scanning involves driving the fast axis with a triangular trajectory and the slow axis with a staircase or ramp signal synchronously [3]. Due to the indigenous lightly damped resonant dynamics, the frequency of the triangular wave is typically restricted within 1 to 10% of the first resonance of these stages, which limits the scanning speed of SPMs significantly [4], [5].

To improve of tracking bandwidth for triangular waves, the general closed-loop feedback control strategies, such as adaptive control [6], sliding-mode control [7], [8], linear quadratic Gaussian (LQG) control [9], as well as negative-imaginary damping controllers, such as integral resonant control (IRC) [10], [11], have been proposed to minimize the residual vibration errors. In spite of significant improvements, the difficulties in tracking high-speed raster scanning remain.

Alternatively, the scanning speed can be increased by implementing nonraster patterns such as Lissajous, cycloid, and spiral methods based on the harmonic waveform at the expense of complex imaging processing in SPMs [4], [12]. In comparison with Lissajous and cycloid scanning patterns, the spiral trajectory can provide a uniform and well-defined image resolution at each point of the scanning area without retracing any part of the targeted area to achieve a higher scanning rate [13]. In the view of controller's design, the primary object for a triangular trajectory is to improve the tracking bandwidth to achieve a fast scanning. Generally, the bandwidth for a triangular trajectory should be approximately ten times the scanning frequency when the first five harmonics are considered. In comparison, the required bandwidth of a spiral scan is approximately 30% lower than the conventional raster scanning [12]. It should be noted that the amplitude of the spiral trajectory increases with tracking time. Therefore, the fast and precision tracking of spiral trajectory with converged error poses new challenges to controller design

in contrast to Lissajous and cycloid trajectories with constant amplitude sinusoids. In [13], a positive position feedback controller was designed to track spiral trajectory with a high closed-loop bandwidth. A multi-input multi-output model predictive control scheme with a damping compensator was proposed to improve tracking speed [14]. In [15], a phase-locked loop-based proportional-integral (PI) controller was implemented to maintain the exact 90° phase shift of the spiral trajectory with the increasing scanning speed. However, the tracking errors with these methods still increase with the varying amplitude so that the scanning performance is significantly deteriorated.

In particular, the internal model controller (IMC) is a practice approach to achieve asymptotic tracking of reference signals with zero steady-state errors [16]. In [17], an IMC has been developed to track a Lissajous trajectory with constant amplitude sinusoids. Moreover, the internal model of a sinusoidal signal was incorporated into LQG controller for spiral scanning [18]. However, the phase error between the input and output is still significant because the built internal model was for constant amplitude sinusoidal signals. To cope with the varying amplitude sinusoidal reference, the IMC synthesized for spiral trajectory was proposed in [19] and [20] to make the tracking error convergent. Note that the nonlinearities such as hysteresis can result in errors in the high-order harmonics of the references [13], [21]. As a consequence, more than three IMC terms that contain high-frequency internal models should be augmented to the final controller, which increases the complexity of parameter computation, and once the frequency of references changes, the controller should be resynthesized.

The conventional repetitive controller (CRC) is also designed based on the internal model principle [16]. The main benefit of this controller lies in that it can provide infinite gains at the fundamental frequency as well as its harmonics of the reference or disturbance [22], [23]. The CRC has been successfully implemented on the piezoelectric nanopositioning stages to track raster trajectory [21], [24], [25]. It is notable that although the tracking errors with CRC can converge to zero with the increasing number of periodic motion, the anticipated reference should be strictly periodic. The CRC is not suitable to track spiral trajectory in spite that it can obtain infinite internal models with a simpler structure and fewer parameters compared with the mentioned IMC. In [26]–[29], a signal transformation approach was proposed and analyzed to improve the steady-state tracking performance while limiting the projected measurement noise. The method transforms the piece-wise linear (affine) references like triangular waveform to ramp signal and designs double-integrator to minimize errors. Although a signal transformation control approach to tracking arbitrary references has been proposed in [30], the reference signal must be continuous and periodic and, thus, is not suitable for the spiral trajectory.

The main objective of this article is to achieve high-speed and precision motion for piezoelectric nanopositioning stages with spiral trajectory. Inspired by the signal transformation approach, a signal-transformation-based repetitive controller (STBRC) is proposed in this article. Compared with IMC in [19] and [20], the proposed method has fewer parameters to be tuned, and is much

easier to implement with spiral trajectories at different frequencies. Furthermore, STBRC also extends the application of CRC that requires strictly periodic signals. The main contribution of this article is three fold. First, the signal transformation approach that converts between the spiral signal and constant amplitude sinusoid, augmented into CRC, i.e., STBRC, is developed to provide infinite gain at the harmonics of the spiral reference. Second, the stability and convergence of steady-state errors of the control method are analyzed with a lifted-system representation for the time-variant property of the controller. Third, the comparative experiments on a piezoelectric nanopositioning stage are performed to elucidate the effectiveness of STBRC on tracking spiral trajectory.

The rest of this article is organized as follows. The background of the spiral trajectory and CRC is showed in Section II. The controller design of STBRC as well as the relevant stability analysis is presented in Section III. Section IV gives the detailed experimental setup. Experiments on a piezoelectric nanopositioning stage and comparisons of the results are elaborated in Section V. Section VI concludes this article.

II. PRELIMINARIES OF SPIRAL TRAJECTORY AND CRC

A. Spiral Trajectory

In this article, the spiral of Archimedes with a constant angular velocity is adopted, which is generated for x - and y -axis by [12]

$$r_x(t) = A(t)\cos(2\pi ft); r_y(t) = A(t)\sin(2\pi ft) \quad (1)$$

where f is the scanning frequency and $A(t)$ is the amplitude varying with time t , which can be expressed as

$$A(t) = pft \quad (2)$$

where p is the pitch, the distance between two consecutive intersections of the spiral curve, can be determined by

$$p = \frac{\text{spiral radius} \times 2}{\text{number of curves} - 1}. \quad (3)$$

From (1), the spiral signal is a sinusoid multiplied by a linearly time-varying signal, which may lead to tracking error increasing with time especially for traditional feedback control, such as built-in PI controller.

B. Conventional Repetitive Controller

The CRC is an effective control algorithm based on internal model principle [16] to provide high gains at the harmonics of the reference trajectory to tracking periodic signals precisely through employing a signal generator $\frac{1}{1-z^{-N}}$, where N is the number of points per period of the reference. Considering a discrete single-input-single-output linear time-invariant (LTI) system $P(z)$ with time-shift operator z , the block diagram of discrete-time CRC with a plug-in implementation is demonstrated in Fig. 1, where $r(k)$ is the reference, $y(k)$ is the output of the plant, $e(k)$ is the tracking error, $u(k)$ is the control force, and $C(z)$, $L(z)$, and $Q(z)$ are the baseline feedback controller (FBC), learning filter, and low-pass filter to improve stability of CRC, respectively. z^{-m} is the compensated time delay caused by the nonminimum-phase characteristic of the piezoelectric

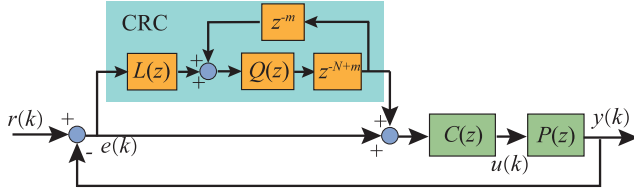


Fig. 1. Block diagram of CRC with a plug-in implementation. In this control scheme, $C(z)$ is a baseline FBC and CRC can be plugged into the control system without changing the exiting $C(z)$.

nanopositioning system. For a minimum-phase system, m can be set as zero. It can be seen in Fig. 1 that the CRC is plugged into the control system without changing the existed baseline FBC, which is easy to be implemented. From Fig. 1, the transfer function of CRC is given by

$$C_{\text{CRC}}(z) = \frac{L(z)Q(z)z^{-N+m}}{1 - Q(z)z^{-N}}. \quad (4)$$

The sensitivity transfer function $S_{\text{CRC}}(z)$ of CRC from $r(k)$ to $e(k)$ is obtained by

$$S_{\text{CRC}}(z) = \frac{1}{1 + [1 + C_{\text{CRC}}(z)]P(z)C(z)}. \quad (5)$$

By substituting (4) into (5) and rearranging terms, it can be deduced that

$$S_{\text{CRC}}(z) = \frac{[1 - Q(z)z^{-N}]S_{\text{fb}}(z)}{1 + Q(z)z^{-N}[z^m L(z)T_{\text{fb}}(z) - 1]} \quad (6)$$

where $S_{\text{fb}}(z)$ and $T_{\text{fb}}(z)$ are sensitivity transfer function and complementary sensitivity function with standalone FBC $C(z)$, i.e.,

$$S_{\text{fb}}(z) = \frac{1}{1 + P(z)C(z)}; T_{\text{fb}}(z) = \frac{P(z)C(z)}{1 + P(z)C(z)}. \quad (7)$$

According to Small Gain Theorem [31], the stability of CRC can be assured if

$$\begin{aligned} & \|Q(z)z^{-N}[z^m L(z)T_{\text{fb}}(z) - 1]\|_{\infty} \\ & = \|Q(z)[z^m L(z)T_{\text{fb}}(z) - 1]\|_{\infty} < 1. \end{aligned} \quad (8)$$

When $r(k)$ is a periodic signal with N samples in one period, $S_{\text{CRC}}(z) = 0$ because of $(1 - z^{-N})R(z)$ within the bandwidth of $Q(z)$, i.e., $|Q(z)|=1$, where $R(z)$ is the z -transformation of $r(k)$. Furthermore, the learning filter $L(z)$, cut-off frequency of $Q(z)$, and delay term m should be chosen properly to retain the stability of CRC. It should be noted that the analysis of the stability condition (8) is conducted in frequency domain that requires the closed-loop system to be LTI.

III. CONTROLLER DESIGN

A. Signal-Transformation-Based Repetitive Control

Inspired by the method in [26], [29], and [30], where the transformation between periodic piece-wise linear reference signals and a ramp reference signal were designed, a mapping between the spiral trajectory and constant amplitude sinusoid is proposed in this article. For a discrete-time system, with the

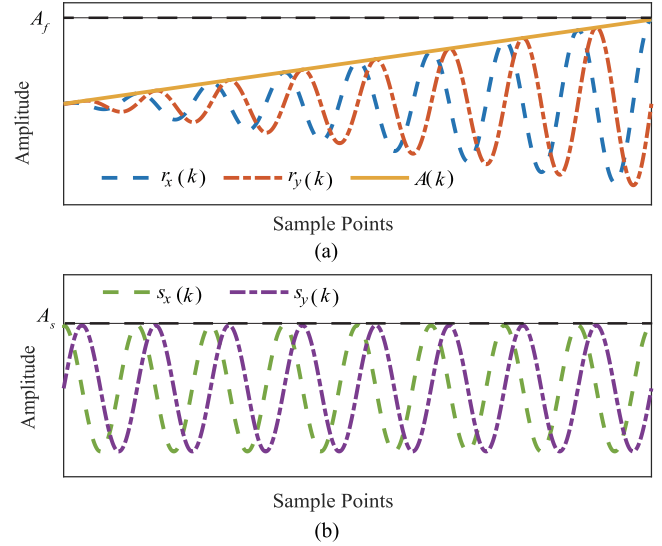


Fig. 2. Illustration of the signal transformation approach for spiral trajectory. (a) Desired references with the maximum amplitude A_f . (b) Transformed signals with the scaled constant amplitude A_s . The amplitude of spiral trajectory $r_x(k)$ and $r_y(k)$ increases with the envelope $A(k)$ and through the $\Phi_{\text{ST}}(k)$ the spiral trajectory is transformed to constant amplitude sinusoid $s_x(k)$ and $s_y(k)$.

signal transformation term $\Phi_{\text{ST}}(k)$ for x - and y -axis, it can be obtained that

$$s_x(k) = A_s \cos(2\pi f T_s k) = \Phi_{\text{ST}}(k) r_x(k) \quad (9)$$

$$s_y(k) = A_s \sin(2\pi f T_s k) = \Phi_{\text{ST}}(k) r_y(k) \quad (10)$$

where T_s is the sample time of the system, A_s is the scaled constant amplitude. Combining with (1) and (2), $\Phi_{\text{ST}}(k)$ can be presented in the follow form:

$$\Phi_{\text{ST}}(k) = \frac{A_s}{pf T_s k}. \quad (11)$$

Similarly, the transformation of constant amplitude sinusoid to spiral trajectory $\Phi_{\text{IST}}(k)$ is derived as

$$\Phi_{\text{IST}}(k) = \frac{pf T_s k}{A_s}. \quad (12)$$

An illustration of the signal transformation approach for a spiral trajectory is demonstrated in Fig. 2. On the one hand, the references of both x - and y -axis are converted to single-tone cosine and sinusoidal waves $s_x(k)$ and $s_y(k)$, respectively, which facilitates the controller design significantly. On the other hand, the original spiral trajectory can also be obtained via $\Phi_{\text{IST}}(k)$. This mapping between spiral trajectory and constant amplitude sinusoid is utilized to develop the proposed controller.

Considering one of motion directions of the piezoelectric nanopositioning stage, the proposed STBRC with a plug-in implementation is illustrated in Fig. 3. The actual input of the system is the scaled constant amplitude cosine or sinusoidal wave that is converted by $\Phi_{\text{ST}}(k)$. The block $\Phi_{\text{IST}}(k)$ between the controller and plant does the reverse action to generate spiral trajectory $y(k)$. As a consequence, $e(k)$ in Fig. 3 is the error caused by the periodic cosine or sinusoidal wave that can converge to zero via employing CRC.

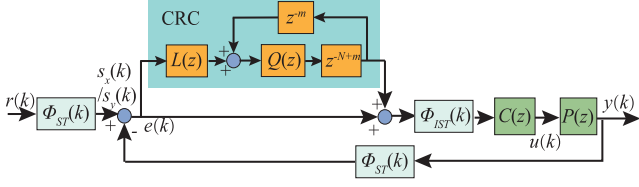


Fig. 3. Block diagram of STBRC with a plug-in implementation for spiral trajectory. Through the implementation of $\Phi_{ST}(k)$, the controller can be designed to track periodic signals and the usage of $\Phi_{IST}(k)$ makes the control force and output convert to the spiral trajectory.

To improve the convergence rate of STBRC, the design of the learning filter $L(z)$ is the inversion of the complementary sensitivity function $T_{fb}(z)$ in general. Note that the piezoelectric nanopositioning stage generally has nonminimum-phase zeros due to the noncollocated actuators and sensors [32]. Therefore, $T_{fb}(z)$ also contains nonminimum-phase zeros that limit the direct application of CRC strategy. In this article, the term z^{-m} is added to adjust the effects caused by the nonminimum-phase characteristic and $L(z)$ is calculated by the zero-magnitude-error tracking (ZMET) method [33]. For the nonminimum-phase plant $T_{fb}(z)$, the transfer function can be decomposed as

$$T_{fb}(z) = \frac{N_s(z)N_u(z)}{D(z)} \quad (13)$$

where N_s and N_u compose of stable and unstable zeros, respectively, and $D(z)$ contains all the stable and identified poles. Besides, N_u can be written as

$$N_u(z) = N_{u,n}z^n + N_{u,n-1}z^{n-1} + \dots + N_{u,0} \quad (14)$$

where n is the number of nonminimum-phase zeros. According to the ZMET method, the approximate inversion of $T_{fb}(z)$, i.e., $L(z)$ is given by

$$L(z) = \frac{D(z)z^{-d}}{N_s(z)N_u^*(z)} \quad (15)$$

where d is the delay steps to make the transfer function casual for practical implementation, and $N_u^*(z)$ is also a n -order polynomial by flipping the coefficients of $N_u(z)$, given by

$$N_u^*(z) = N_{u,0}z^n + N_{u,1}z^{n-1} + \dots + N_{u,n}. \quad (16)$$

It can be seen that for the nonminimum-phase plant, the learning filter always leads to delay in the error signal $e(k)$. Hence, an adjustable term m is added to STBRC design to achieve the anticipated performance.

It should be mentioned that it is not possible to establish a perfectly accurate model $P(z)$ in practice. To improve the robustness of the proposed controller to cope with model uncertainties in the high-frequency region, a zero-phase low-pass filter is employed as

$$Q(z) = az + b + az^{-1} \quad (17)$$

and a, b should satisfy $2a + b = 1$ to ensure a unit gain below the cut-off frequency [34]. Although $Q(z)$ improves the robustness, the tracking performance is deteriorated, especially for high-frequency errors. A tradeoff between stability and tracking

accuracy should be taken into consideration when selecting a and b .

Note that the nonlinear hysteresis effect is treated as external disturbances in this article. Generally, this nonlinearity generates errors with the infinite number of harmonics in addition to its fundamental component of the reference [17], [21]. Therefore, through the signal transformation, the hysteresis can also be mitigated through the proposed STBRC.

B. Stability and Performance Analysis

In this section, the stability and tracking performance are analyzed. For a clear expression, all the vectors are written as regular, lowercase, bold letters with the length N , and the matrices are regular, capital, bold letters with the dimension of $N \times N$.

Note that the time-variant terms $\Phi_{ST}(k)$ and $\Phi_{IST}(k)$ are included in the STBRC strategy so that the stability criterion (8) is not applicable to this article. Inspired by the time-domain analysis in iterative learning control [35], a lifted-system framework is adopted to deduce the stability of STBRC in this article. To construct the lifted-system representation, the plant $P(z)$ can also be expressed as a state-space model, i.e.,

$$\begin{aligned} \mathbf{x}_p(k+1) &= \mathbf{A}\mathbf{x}_p(k) + \mathbf{B}\mathbf{u}(k) \\ y(k) &= \mathbf{C}\mathbf{x}_p(k) + \mathbf{D}\mathbf{u}(k) \end{aligned} \quad (18)$$

where $\mathbf{x}_p(k)$ is the state vector of $P(z)$ and $\mathbf{A}, \mathbf{B}, \mathbf{C}$, and \mathbf{D} are the state matrix, input matrix, output matrix, and feedthrough matrix, respectively. For a CRC system, the dynamics can also be deemed over the finite periods with N samples. Hence, the dynamics on the j th period of $P(z)$ are described in the equivalent lifted form as

$$\mathbf{y}(j) = \mathbf{P}\mathbf{u}(j) \quad (19)$$

where j is the current number of period and

$$\mathbf{P} = \begin{bmatrix} \mathbf{D} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{CD} & \mathbf{D} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{CA}^{N-2}\mathbf{D} & \mathbf{CA}^{N-3}\mathbf{D} & \dots & \mathbf{D} \end{bmatrix} \quad (20)$$

$$\mathbf{u}(j) = [u(jN-N) \ u(jN+1-N) \ \dots \ u(jN-1)]^\top \quad (21)$$

$$\mathbf{y}(j) = [y(jN-N) \ y(jN+1-N) \ \dots \ y(jN-1)]^\top \quad (22)$$

where $\mathbf{u}(j)$ and $\mathbf{y}(j)$ are constructed by $u(k)$ and $y(k)$ with $k \in \{jN-N, jN+1-N, \dots, jN-1\}$. When a delay is contained in the plant, a slight change of (20)–(22) should be made to enhance the similarity of the frequency-domain model [35].

To analyze the stability of STBRC, the block diagram of STBRC with a lifted-system representation at period j is presented in Fig. 4, where $\mathbf{G}_d, \mathbf{L}, \mathbf{Q}$ are the lifted matrices of $P(z)C(z), L(z)$, and $Q(z)$, respectively. \mathbf{M}^{-N+m} and \mathbf{M}^{-m} are the lifted representation of terms with $-N+m$ and m -step

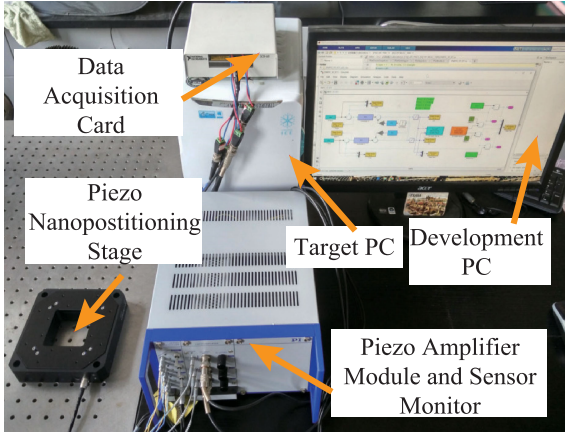


Fig. 5. Experimental setup of the piezoelectric nanopositioning stage.

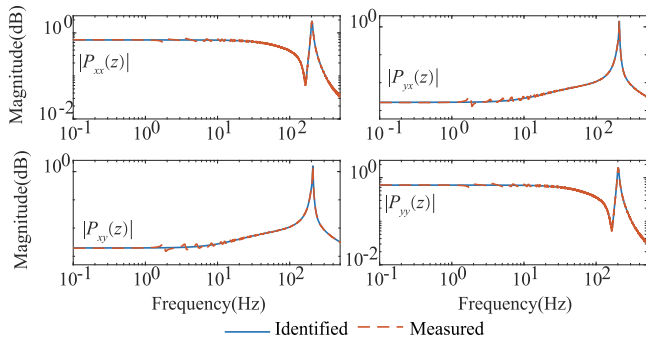


Fig. 6. Identified and measured amplitude responses of the stage.

control system is built in Simulink real-time environment on the host computer and executed real-time on the target computer. In order to identify the linear dynamic model of the piezoelectric nanopositioning stage, a sine-sweep input with a constant low amplitude with the frequencies between 0.1 and 500 Hz is applied to the x - and y -axis. After being discretized via zero-order holder method, the linear discrete transfer functions of x to x and y to x are given as

$$P_{xx}(z) = \frac{0.014z^4 - 0.054z^3 + 0.123z^2 - 0.128z + 0.056}{z^5 - 3.82z^4 + 6.316z^3 - 5.54z^2 + 2.58z - 0.51} \quad (38)$$

$$P_{xy}(z) = \frac{0.002z^4 - 0.009z^3 + 0.016z^2 - 0.013z + 0.004}{z^5 - 4.016z^4 + 7.141z^3 - 6.87z^2 + 3.57z - 0.81} \quad (39)$$

The first resonant frequency of both x - and y -axis is 210 Hz from Fig. 6, which limits the motion within a low speed when implementing the stand-alone FBC. In addition, there exists cross-coupling effects according to Fig. 6 and it is especially obvious near the first resonant frequency. In this article, the cross-coupling error is considered as the external disturbance to simplify the implementation of the proposed controller. Note that although the nanopositioning stage cannot achieve video-rate imaging, the proposed controller can also be implemented on a stage with a higher resonant frequency.

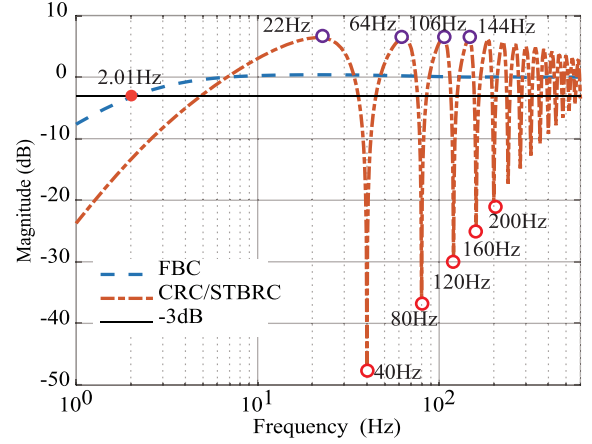


Fig. 7. Sensitivity transfer functions of the baseline FBC and CRC/STBRC for x -axis with a 40 Hz reference.

B. Controller Implementation

For the baseline FBC, a notch filter with an integral controller is designed primarily [36], where the notch filter's characteristic frequency is designed at 203 Hz to avoid the zero-pole cancellation as well as suppress the effect of the resonant peak, and a pair of poles are assigned at 2000 Hz to attenuate high-frequency noise. Furthermore, to improve the tracking performance, an integral gain 300 is cascaded before the notch filter. To facilitate the design of STBRC, the FBC is obtained by discretizing via bilinear transformation method as

$$C_x(z) = C_y(z) = \frac{0.0103z^3 - 0.0062z^2 - 0.0065z + 0.01}{z^3 - 2.111z^2 + 1.963z - 0.8519} \quad (40)$$

For the design of CRC and STBRC, the learning filters are calculated by the ZMET method using (15), and $a = 0.25$, $b = 0.5$ are chosen for the zero-phase low filter $Q(z)$. Because the nanopositioning stage is a nonminimum-phase system, the delay term m is determined as 3 for both x - and y -axis for the symmetric structure of the stage. The sensitivity transfer functions of x -axis for FBC, CRC, and STBRC are plotted in Fig. 7. In this article, the FBC in the proposed controller is designed with a sufficiently low bandwidth at 2.01 Hz to evaluate its advantage to tracking the high-speed trajectory and suppressing low-frequency disturbance, such as cross coupling and attenuate noises. Taking a 40 Hz reference as an example, STBRC can provide multiple lower gains at the fundamental frequency and its harmonics to ensure zero steady-state errors. It should be noted that when implementing CRC, $\Phi_{ST}(k)$ and $\Phi_{IST}(k)$, given as (11) and (12), are set as 1 and for STBRC.

In this article, a zero-phase low-pass filter is adopted to ensure the robustness of STBRC. Hence, \mathbf{Q} in (31) is a symmetric matrix instead of a lower triangular one. In spite of that, the stability condition is still correct for $\rho(\mathbf{E}) \approx \rho[\mathbf{Q}(\mathbf{I} - \mathbf{L}\mathbf{T}_{fb})]$, which can be concluded in Tables I and II, and the maximum eigenvalues without transformation terms are 0.891 and 0.887, respectively. Therefore, (32) can also be employed to ensure the stability and error convergence of STBRC with a zero-phase filter for spiral trajectory. Besides, the maximum eigenvalues in

TABLE I
MAXIMUM EIGENVALUE $\rho(\mathbf{E})$ AT DIFFERENT PERIODS FOR
40 HZ SPIRAL TRAJECTORY OF x -AXIS

j	2	5	15	20	25
$\rho(\mathbf{E})$	0.888	0.889	0.890	0.890	0.891

TABLE II
MAXIMUM EIGENVALUE $\rho(\mathbf{E})$ AT DIFFERENT PERIODS FOR
50 HZ SPIRAL TRAJECTORY OF y -AXIS

j	2	5	15	20	25
$\rho(\mathbf{E})$	0.885	0.886	0.887	0.887	0.887

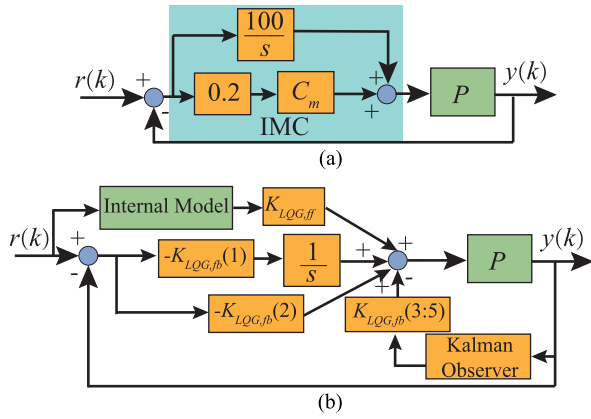


Fig. 8. Block diagram of other controllers in this article. (a) IMC. (b) IMLQG.

different periods are nearly the same, which demonstrates that the time-variant transformation terms have no influence on the stability.

V. COMPARATIVE EXPERIMENTS

A. Comparison With Internal Model Based Controllers

The IMC is also an effective method to track a known signal. In this section, an IMC in [19] and [20] based on H_∞ mixed-sensitivity synthesis [37] is designed to compare with the proposed STBRC. A stable sensitivity weight $W_S(s) = [1 + \frac{2 \times 10^{-4} s}{2\pi f} + (\frac{s}{2\pi f})^2]^{-2}$ and constant control weight $W_U(s) = 5$ are used in MATLAB function mixsyn. After reducing the order, the resulting IMC for 40 Hz spiral trajectory may be written as

$$C_m(s) = \frac{0.27 s^4 - 82 s^3 + 25370 s^2 - 7.9 \times 10^6 s - 2.5 \times 10^9}{s^4 + 0.1 s^3 + 1.26 \times 10^5 s^2 + 6350 s + 3.99 \times 10^9}. \quad (41)$$

An integral controller is also added to improve the steady-state precision, and the block diagram of this controller is implemented as given in Fig. 8(a). The final IMC controller is determined as

$$C_{\text{IMC}}(s) = \frac{100}{s} + 0.2C_m(s). \quad (42)$$

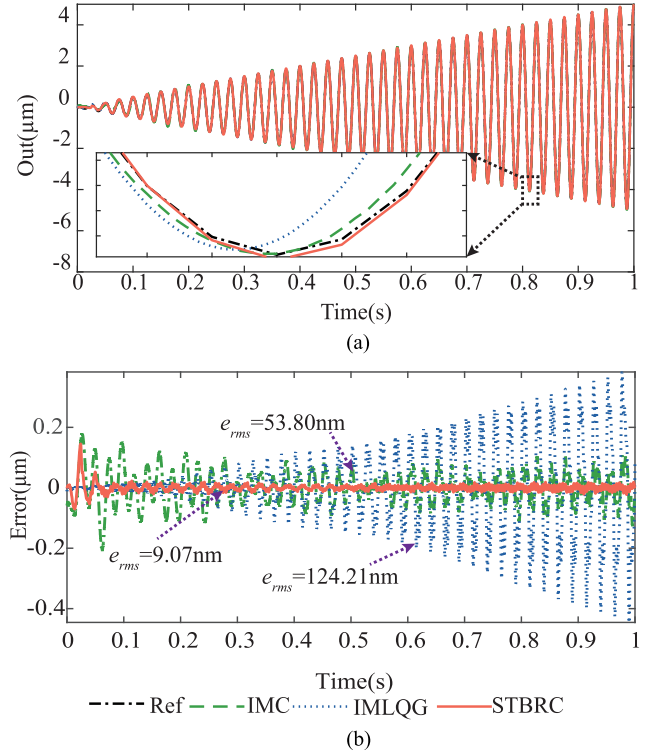


Fig. 9. Experimental results of IMC, IMLQG, and proposed STBRC of x -axis at 40 Hz spiral. (a) Spiral trajectory tracking. (b) Tracking errors.

Besides, an internal-model-based LQG controller (IMLQG) proposed in [18] is also designed as a benchmark in this article with block diagram in Fig. 8(b). In this method, the internal model of the reference sinusoidal signal is combined into the plant model, and an integrator is introduced with an LQG controller to improve the tracking performance. Additionally, a Kalman observer with the variance of 0.002 and covariance matrix $10^{-6} \mathbf{I}_{5 \times 5}$ is employed to calculate the observer vector, and subsequently estimate the states of the plant in order to implement the state feedback control [20]. In this article, the weighting matrices are selected as $Q_{\text{LQG}} = \text{diag}[10^7 \ 0.008 \ 1 \ 1 \ 1]^T$ and $R_{\text{LQG}} = 0.01$, respectively. Then, the feedback and feedforward gains are calculated as

$$\begin{aligned} K_{\text{LQG,fb}} &= [-3162.2 \ -38.8 \ 2.7 \ 143.6 \ 304.3 \ 268.2] \\ K_{\text{LQG,ff}} &= [-17.7 \ 3393.5]. \end{aligned} \quad (43)$$

The tracking results of 40 Hz spiral trajectory for internal model based controllers with a maximum $10 \mu\text{m}$ peak-to-peak amplitude are demonstrated in Fig. 9. Both the IMC and STBRC can achieve convergence of the steady-state errors as the increase of spiral amplitude, while the error of IMLQG increases with the amplitude with the root-mean-square errors (e_{rms}) 124.21 nm. This is because the model of the varying amplitude is not taken consideration into IMLQG [20]. In this article, only the fundamental frequency internal model at 40 Hz is built in (42) so that the e_{rms} is 53.80 nm in contrast to 9.07 nm with STBRC. The results show that the STBRC achieves the better performance than the IMC. The better performance can also

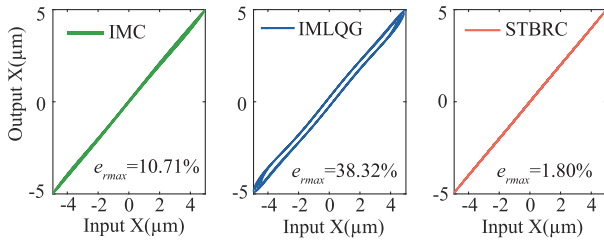


Fig. 10. Experimental results of the relations between actual displacements and the desired reference of x -axis at 40 Hz spiral.

be obtained by IMC with more high-frequency internal models contained in (42) at the cost of more parameters to be tuned, especially for different tracking frequencies. In comparison, the only parameter N is needed to be adjusted along with tracking frequency in the premise of stability condition (31).

To investigate the effect of hysteresis nonlinearity on the controllers, the results of the relations between the actual displacements and the desired reference are plotted in Fig. 10. The relative maximal errors (e_{rmax}) of the hysteresis loops for different controllers are 10.71%, 38.32%, and 1.80%, respectively, which demonstrates the proposed STBRC can mitigate hysteresis phenomenon effectively compared with other internal model based controllers.

B. Spiral Trajectory Tracking

1) *Suppression of Cross-Coupling Effect*: In this article, the controllers are designed separately for x - and y -axis by taking cross-coupling effects as disturbances. To validate the effectiveness of STBRC on suppression of cross-coupling errors, a 50 Hz spiral trajectory with a maximum 10 μm peak-to-peak amplitude is injected into the x -axis of the stage. It is clear that the cross-coupling errors increase with the amplitude and is drifting with time for open-loop (OL) result in Fig. 11. e_{rms} is 3.79 nm and 10.92 nm for standalone FBC and IMLQG, while CRC and STBRC achieve lower e_{rms} with 2.46 and 2.41 nm, respectively. The results show that the proposed controller can compensate for cross-coupling effects substantially.

2) *High-Speed Tracking Results*: The tracking performance of the proposed controller is tested with a spiral trajectory with a maximum 10 μm peak-to-peak amplitude at 40 and 50 Hz, respectively. The tracking results of XY spiral are demonstrated in Figs. 12 and 13. It can be concluded that both the OL and FBC have less scanning areas than the references (Ref) for the lower gains and higher delay at closed-loop high-frequency regions. The tracking errors of 40 Hz, 50 Hz for x - and y -axes are given in Figs. 14 and 15. It is evident that STBRC can make the steady-state error convergent after several circles through learning the last circle's error while the error of CRC remains the same with the increase of reference amplitude. This is because the error in the previous period is not repetitive in the current period, resulting in less control force of CRC. The error of IMLQG increases with time although its performance is better than OL and FBC.

The e_{rms} and maximal errors (e_{max}) are tabulated in Tables III and IV in detail. It should be noted that the error of FBC is larger

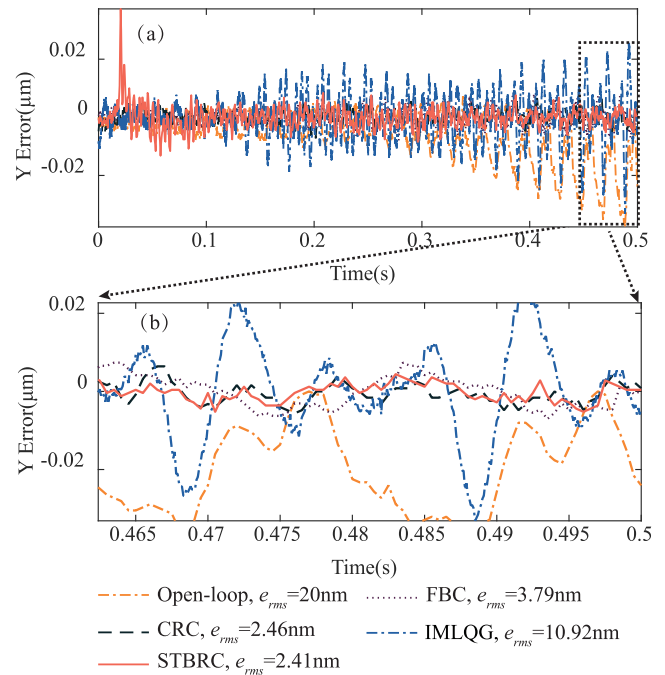


Fig. 11. Experimental results of cross-coupling effect for y -axis at 50 Hz spiral. (a) Cross-coupling errors of y -axis. (b) Zoomed-in view of the tracking errors.

TABLE III
STATISTICAL RESULTS OF STEADY-STATE TRACKING ERRORS WITH 40 HZ SPIRAL TRAJECTORY (UNIT:NM)

Statistical Errors(nm)	OL	FBC	CRC	IMLQG	STBRC
x axis e_{rms}	2602.1	4137.2	118.43	249.89	6.42
e_{max}	4071.5	6311.2	170.53	410.39	17.47
y axis e_{rms}	2658.6	4173.2	112.49	240.42	6.51
e_{max}	3946.3	6247.1	158.40	370.44	15.21

TABLE IV
STATISTICAL RESULTS OF STEADY-STATE TRACKING ERRORS WITH 50 HZ SPIRAL TRAJECTORY (UNIT:NM)

Statistical Errors(nm)	OL	FBC	CRC	IMLQG	STBRC
x axis e_{rms}	2946.5	3923.1	148.72	318.95	6.46
e_{max}	4702.7	6140.7	209.53	486.22	15.84
y axis e_{rms}	3000.6	3918.9	139.89	346.44	5.64
e_{max}	4510.8	5978.3	198.90	484.90	13.19

than that of OL due to the sensitivity bandwidth of the designed FB controller of 2.01 Hz in order to suppress disturbances and reduce projection noises. For the tracking performance of STBRC, e_{rms} and e_{max} of x at 50 Hz are 6.46 and 15.84 nm, respectively, in comparison with 148.72 and 209.53 nm of CRC, 318.95 and 486.22 nm of IMLQG. Also note that when the tracking frequency varies from 40 to 50 Hz, the statistical errors of STBRC are all within 6.5 and 18 nm, i.e., 0.065% and 0.18% of the maximum amplitude, respectively. A similar conclusion can also be drawn from the tracking performance of y -axis, which validates the effectiveness of STBRC. Besides, the hysteresis loops of different controllers at 50 Hz spiral with steady state

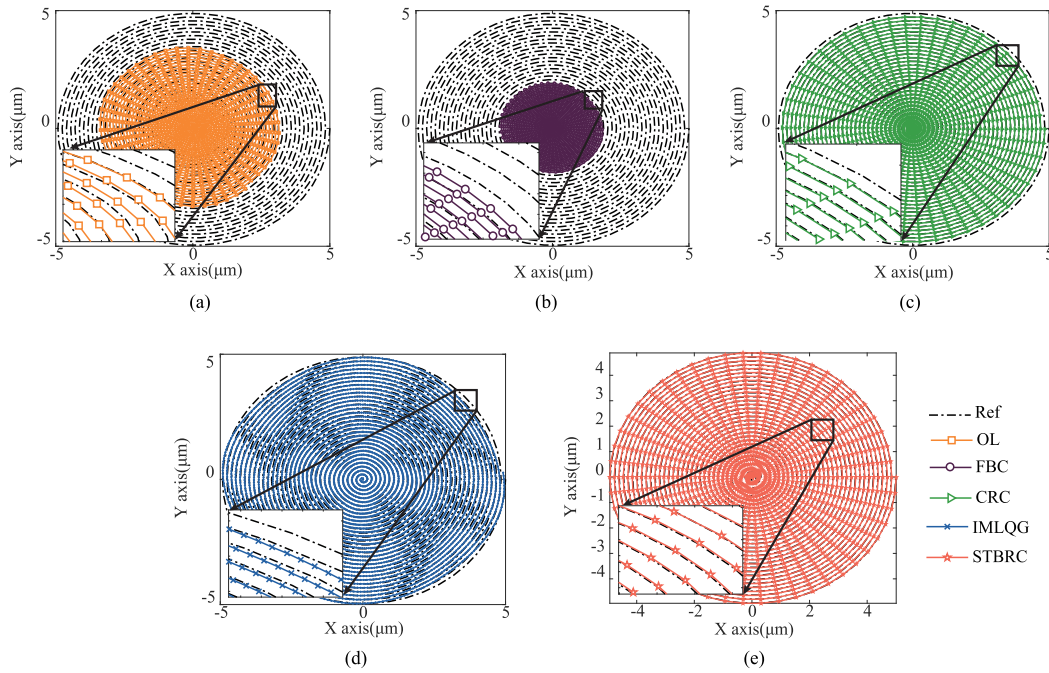


Fig. 12. Tracking results of different controllers for 40 Hz spiral trajectory. (a) OL. (b) FBC. (c) CRC. (d) IMLQG. (e) STBRC.

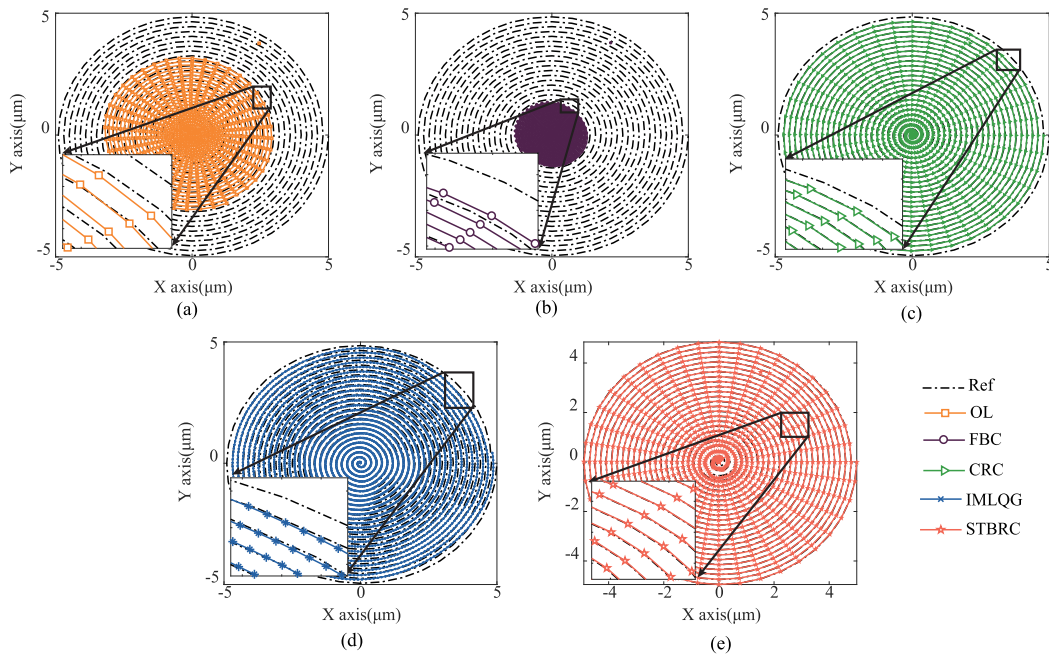


Fig. 13. Tracking results of different controllers for 50 Hz spiral trajectory. (a) OL. (b) FBC. (c) CRC. (d) IMLQG. (e) STBRC.

are also given in Fig. 16, where the e_{rmax} of STBRC achieves 1.60%, showing the best performance among these controllers.

Note that in this section, two scenarios with 40 and 50 Hz spiral trajectories are tested to show the flexibility and adaptability of the proposed method at different frequencies primarily. The further improvement of the tracking speed on the nanopositioning stage in Fig. 5 is limited by the mechanical design and driving principle, and the system in this article is employed to verify the

effectiveness of the proposed controller on spiral tracking. The disturbances or uncertainties within the unmatched frequency regions as shown in Fig. 7 also deteriorate the performance of a higher speed tracking. In spite of this, the tracking frequency of 50 Hz reaches 23.81% of the first resonant frequency at 210 Hz, which achieves the best performance among the values of 21.57% in [15], 17.57% in [18] with piezoelectric actuator under fair comparisons.

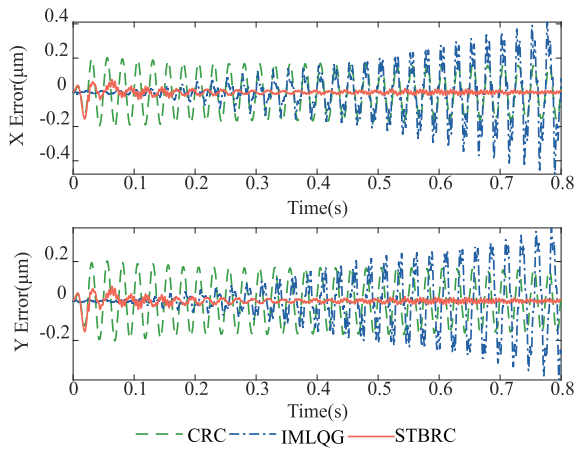


Fig. 14. Tracking errors of 40 Hz trajectory for x - and y -axis.

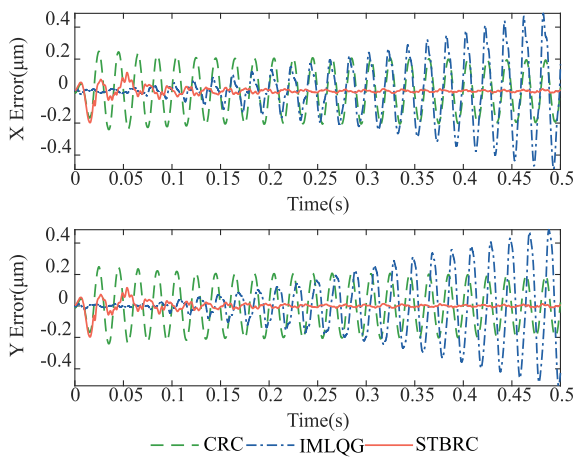


Fig. 15. Tracking errors of 50 Hz trajectory for x - and y -axis.

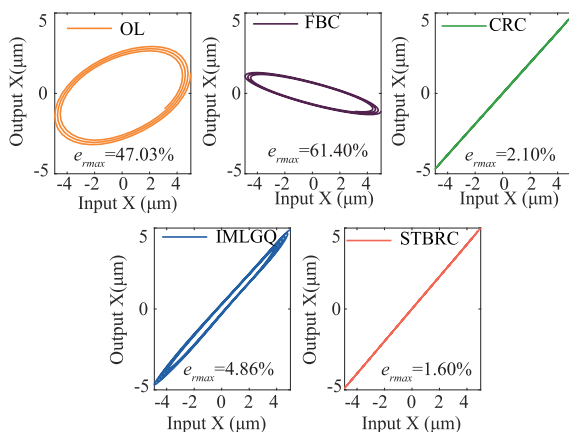


Fig. 16. Experimental results of the relations between actual displacements and the desired reference of x -axis at 50 Hz spiral with steady state.

VI. CONCLUSION

In this article, the STBRC was proposed for high-speed and high-precision tracking of spiral trajectory. Different from the signal transformation approach to map between piecewise reference signal and ramp signal, a transformation for spiral

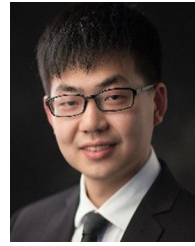
trajectory to constant amplitude sinusoid was designed. Through integrating with CRC, the STBRC can provide infinite internal models of spiral trajectory, even at high-frequency harmonics, to achieve convergent steady-state errors. The proposed method extends the application of CRC that requires strictly periodic signals. Due to the time-variant property of the overall system, the stability and error convergence of the STBRC is proven in a lifted-system representation. The experiments on a piezoelectric nanopositioning stage were also performed. Comparative results demonstrated that the proposed controller achieves the better performance than internal model based controllers with a simpler structure and fewer parameters. Furthermore, the cross-coupling e_{rms} was suppressed to 2.41 nm, and e_{rms} and e_{max} of both x -axis and y -axis are below 0.065% and 0.18% of the maximum amplitude for high-speed spiral tracking, showing the best performance through comparisons.

Despite that the proposed method can handle low-frequency disturbances effectively, the controller may amplify the disturbances that do not match with the fundamental frequency and its harmonics of reference, as is shown in Fig. 7. The future work will concentrate on dealing with this by employing disturbance observer technology.

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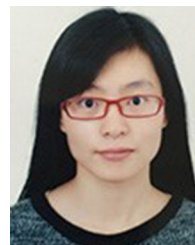
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